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THE APPLETON ARITHMETICS



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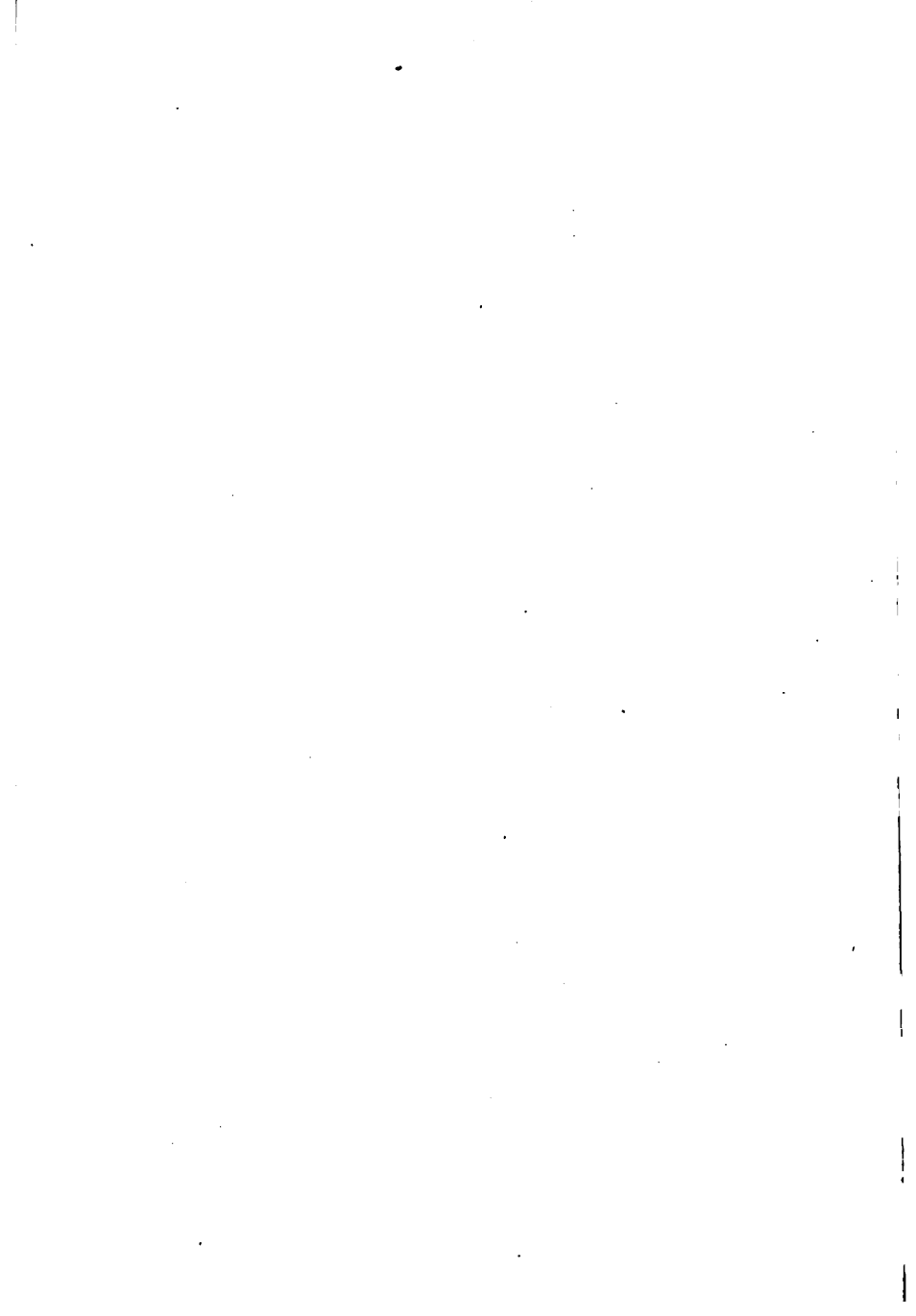
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THE APPLETON ARITHMETICS

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THIRD BOOK



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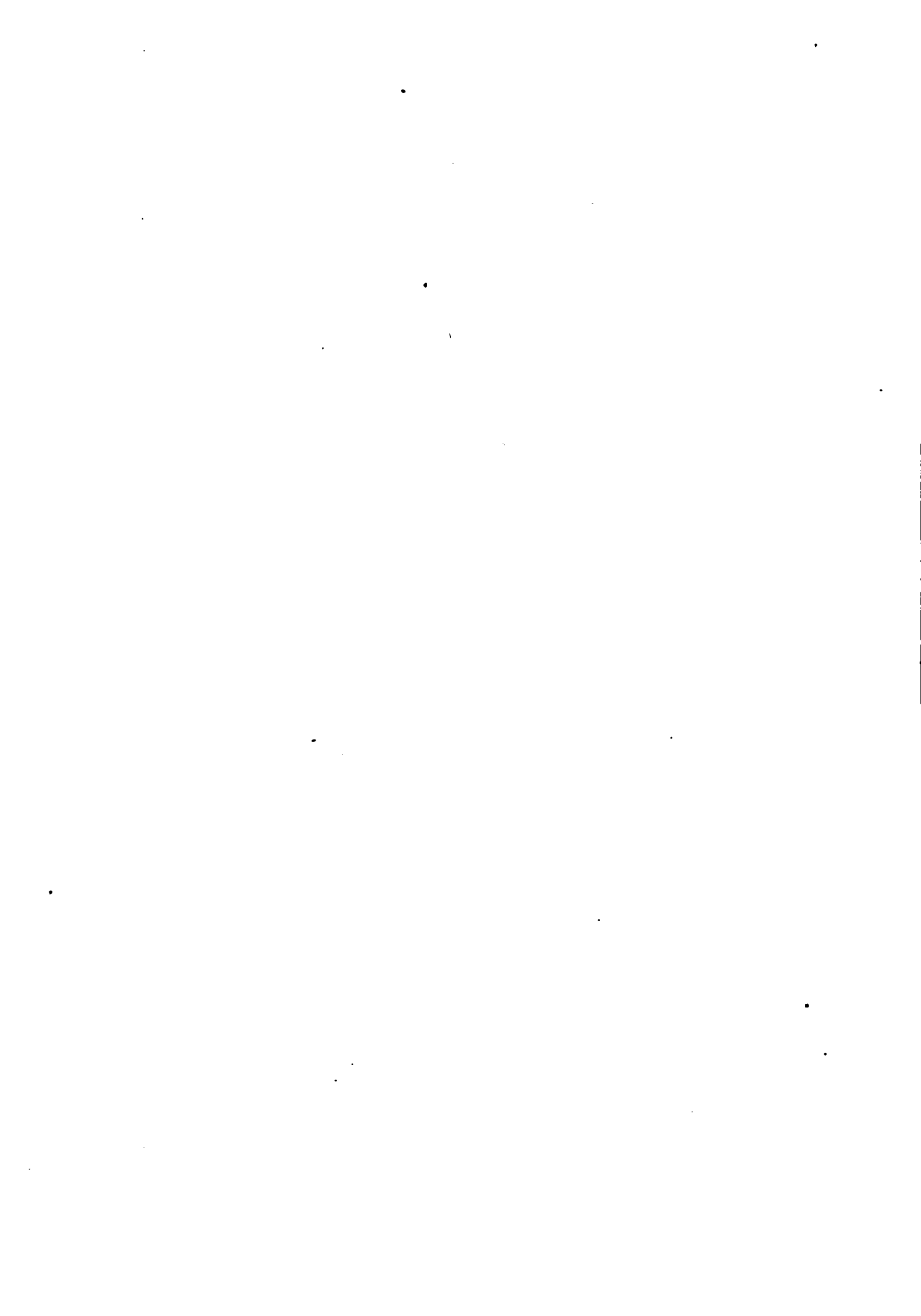
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P R E F A C E

THE present volume of THE APPLETON ARITHMETICS covers the work usually taught in the seventh and eighth grades.

Like the two preceding books, it is a text for the *pupils'* use and, with suitable modifications, continues to use the mode of presentation of its predecessors in the series. It reviews and summarizes the theory of arithmetic, furnishes more difficult drill, extends its applications into advanced business arithmetic, makes use of the equation in the solution of arithmetical problems, and introduces such algebra, geometry, and measurement as prepare for high-school work.

THE AUTHORS.



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THIRD BOOK

CHAPTER I

FUNDAMENTAL PROCESSES WITH INTEGERS AND DECIMALS

NOTATION AND NUMERATION

1. The method of writing numbers is called **notation**, and that of reading them **numeration**.

2. Two **systems of notation** have been explained: The **Arabic** or **Hindu** system of notation, with the symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9;

and the **Roman** system, with the symbols:

I for 1.	X for 10.	C for 100.	M for 1,000.
V for 5.	L for 50.	D for 500.	

The Roman numbers from 1 to 10 are written:

I, II, III, IV, V, VI, VII, VIII, IX, X.

In Roman numerals the multiples of 10 from 10 to 100 are written:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC, C.

3. For convenience in reading, numbers are set off into **periods** of three digits each. The last period at the left may have fewer than three digits.

A comma (*separatrix*) or a larger space than that between the other figures may be used to separate two periods.

Thus, 8,569,240,007 or 8 569 240 007.

4. The first four periods of integers are named **units, thousands, millions, billions**.

Thus, \$2,630,400,521 is read: *Two billion six hundred thirty million four hundred thousand five hundred twenty-one dollars.*

5. The decimal places in order to the right from the decimal point are named: **tenths, hundredths, thousandths, ten thousandths**, etc.

The decimal is read as if it were an integer, then the name of the last order is added.

Thus, .0256 is read: *Two hundred fifty-six ten-thousandths.*

Decimal places beyond the fourth are rarely used.

In an integer the decimal point belongs at the right, but it is seldom written.

6. The following table shows the names of the periods and places from millionths to billions:

Hundred-billions	Hundred-millions	Hundred-thousands	Hundreds	Ten-thousands	Ten-thousandths	Ten-thousandths
Ten-billions	Ten-millions	Ten-thousands	Tens	Thousands	Hundredths	Hundred-thousandths
Billions	Millions	Thousands	Units	Hundreds	Thousandths	Millionths
5	2	6	5	8	0	7
0	3	8	2	0	0	4
7	7	0	1	0	2	5
Billions	Millions	Thousands	Units	Thousandths	Millionths	

Numbers of more than four periods are very rarely needed. The three periods next in order are called, in the system most used, **trillions, quadrillions, quintillions**.

ORAL EXERCISES

1. Read the number in the table above. How many periods are there in the integral part? In the decimal part? How many orders in each period?

2. Read the numbers in the table showing the number of pounds of freight that passed through the various waterways in a recent year:

WATERWAYS.	POUNDS.	TONS.
Suez.....	8,560,284,000	()
Welland.....	869,596,000	()
Sault Ste. Marie.....	15,062,594,000	()
Detroit River.....	25,845,696,000	()

3. State the number of tons, or the tonnage, for each waterway named in Exercise 2.

Read:

4. Some light waves are .00011 in. long; others are .00001 in. long.

5. Sea water is 1.0249 times as heavy as fresh water.

6. The length of a circle is nearly 3.14159 times the length of its diameter.

7. Light waves whose lengths are greater than .000029 in. or less than .000013 in. are not visible to the eye.

8. In a recent year some of the imports and exports of the United States were:

IMPORTS.	EXPORTS.
Sugar..... 3,031,915,875 lb.	Cotton..... 3,500,778,763 lb.
Coffee..... 1,091,004,252 lb.	Wheat..... 154,856,102 bu.

9. The yearly earnings of the national banks of the United States for eight years were:

(1)..... \$87,276,836	(5)..... \$112,936,426
(2)..... 81,853,797	(6)..... 105,909,385
(3)..... 106,581,476	(7)..... 127,526,836
(4)..... 109,881,530	(8)..... 152,235,434

WRITTEN EXERCISES

Write in figures :

1. Two million, eighty-three thousand, seventeen.
2. Five million, one hundred six thousand, two hundred.
3. Seventeen million, three hundred thousand, six hundred twenty-five.
4. Three hundred seventy-five thousand four hundred six and one hundred twenty-five thousandths.
5. Six billion, eight hundred five million, four hundred fifty-six thousand, two hundred seventy-nine.

Write the following numbers in words :

- | | |
|---------------------|------------------------|
| 6. 3,547,000.275. | 11. 126, 379, 008. |
| 7. 6,000,001.33. | 12. 96, 439, 109, 100. |
| 8. 11,111,101.111. | 13. 436, 701.005. |
| 9. 22,202,200.002. | 14. 19,000,000.000019. |
| 10. 77,000,070.077. | 15. 125, 000.125. |
16. Write 200 million in figures. 400 million. 790 million.
17. Write 1000 million in figures. How many periods are there in the number? What is the name of the fourth period? Read the number.

ADDITION

7. Addition is the process of counting two or more numbers into an equivalent single number.

ORAL EXERCISES

1. What is the result of addition called?
2. What are the numbers added called?
3. When numbers are written for adding, how should the corresponding orders stand?
4. How may the work of addition be tested?

WRITTEN EXERCISES

1. Add 200, 1,760, 25.7, and 2.005. Also. 16.025, 30.008, 9.2693, and .0011.

2. Find from the table below the total number of people working in the lumber business in the United States in a certain year:

In furniture factories....	58,719	In saw mills.....	100,087
" cooper shops.....	37,226	" planing mills.....	61,600
" piano factories.....	6,220	" other factories.....	82,390

3. Find the total amount of receipts of a school system from the following school funds:

Instruction fund.....	\$440,976.12
Building fund.....	120,000.00
Repair fund.....	25,046.75
Library fund.....	5,138.70
Contingent fund.....	144,411.50

4. Recently the governmental departments of a certain city had on hand the following sums:

Department of education.....	\$81,000.00
Department of public works.....	69,374.62
Department of charities and correction.....	24,600.00
Department of public safety.....	21,817.92
Legislative department.....	12,000.00
Department of law.....	8,000.00
Comptroller's office (estimated).....	5,000.00
Treasurer's office.....	3,000.00
Department of assessment and taxation.....	395.00
Fire marshal.....	400.00
Civil service commission.....	300.00
Park commission.....	300.00
Mayor's office.....	300.00
Municipal court.....	25.00
Police court.....	25.00

Find the total amount on hand.

Casting Out Nines

8. Dividing the sum of the digits of a number by 9 is called **casting out the nines**.

EXAMPLE: Add the digits in the number 2,639. Divide the sum by 9. The remainder, 2, is the excess.

The remainder after the 9's are cast out is called the **excess**.

ORAL EXERCISES

Cast out the nines from the following numbers.

- | | | |
|-------------|------------|-----------------|
| 1. 40,372. | 5. 3,061. | 9. 72,320. |
| 2. 56,432. | 6. 3,136. | 10. 125,963. |
| 3. 172,360. | 7. 9,910. | 11. 1,265,384. |
| 4. 125,678. | 8. 14,037. | 12. 69,327,436. |

9. Addition may be tested by casting out **nines**.

(1) EXCESS.

26731	1
37654	7
<u>64385</u>	8

In the first example the sum of the excesses of the *addends* is 8, and the excess of the *result* is 8; hence the work checks.

(2)

2635	7
3094	7
<u>4265</u>	8
9994	4

In the second example the sum of the excesses is 22, whose excess is 4. The excess of the result, 9,994, is 4. Hence the work checks.

The process is the same when the addends contain a decimal part.

WRITTEN EXERCISES

Add, and test the results by casting out nines:

- | | | | |
|--------------|--------------|--------------|---------------|
| 1. 63450 | 3. 76327 | 5. 26932 | 7. 260032 |
| 85709 | 94308 | 26872 | 310032 |
| <u>36245</u> | <u>86347</u> | <u>26462</u> | <u>187625</u> |
| 2. 8275 | 4. 1403 | 6. 1127 | 8. 4083 |
| 3268 | 3399 | 8943 | 986 |
| <u>7457</u> | <u>2401</u> | <u>1794</u> | <u>77</u> |

SUBTRACTION

7

9. 180060	11. 163.006	13. 347798	15. 198.221
19760	29.376	543297	768.993
4330	197.263	936720	231.795
<u>308699</u>	<u>90.008</u>	<u>399444</u>	<u>550.117</u>
10. 12.7	12. 40.30	14. 90.80	16. 3.125
14.3	17.25	12.60	0.625
40.8	16.83	30.17	3.315
<u>90.6</u>	<u>40.36</u>	<u>45.96</u>	<u>8.055</u>

17. The following are the approximate amounts of the contracts for constructing the Williamsburgh Bridge from Manhattan to Brooklyn:

For the Manhattan tower foundation.....	\$373,462.70
" " Brooklyn "	485,082.70
" " Manhattan anchorage.....	797,770.00
" " Brooklyn "	771,778.00
" " Manhattan approach.....	1,464,000.00
" " Brooklyn "	947,000.00
" " suspended structure.....	1,123,400.00
" " steel towers and end spaces.....	1,221,726.00
" " minor work.....	1,500,000.00

Find the total cost of the bridge. Test the accuracy of your work by casting out nines.

SUBTRACTION

10. Subtraction is the process of finding how much one number exceeds another.

ORAL EXERCISES

1. What is the result of subtraction called?
2. In a case of subtraction what is the number subtracted called? What is the number from which another is taken called?
3. How is the work of subtraction tested?

4. When numbers are subtracted, how are their corresponding orders written? When decimal numbers are subtracted, how are the decimal points made to stand?

WRITTEN EXERCISES

Subtract and test :

1. 7893 <u>2469</u>	4. 8654.23 <u>799.86</u>	7. 5002.17 <u>673.43</u>	10. 1000.00 <u>.99</u>
2. 84.33 <u>23.99</u>	5. 126.003 <u>46.098</u>	8. 909.12 <u>508.20</u>	11. 2000.00 <u>9.87</u>
3. 8703 <u>2604</u>	6. 763.45 <u>97.23</u>	9. 406.38 <u>289.</u>	12. 1111.110 <u>99.992</u>

Solve :

13. The value of the clay pottery made in a recent year was:			
Ohio.....	\$10,519,138	West Virginia.....	\$1,166,464
New Jersey.....	6,192,959	All other States.....	(?)
Pennsylvania.....	1,876,263	United States.....	24,127,453

What was the total value of the pottery produced in Ohio, New Jersey, Pennsylvania, and West Virginia? In all other States?

14. The following table shows the loans and deposits for certain New York banks for one week:

NAME OF BANK.	TOTAL LOANS.	SHORT-TIME LOANS.	DEPOSITS.
Liberty National.....	\$12,795,400	\$2,401,500	\$10,976,800
N. Y. Produce Exchange....	6,250,400	1,895,900	7,533,500
New Amsterdam National...	4,447,400	678,200	4,945,600
State Bank.....	14,290,000	2,223,000	15,179,000
Fourteenth Street Bank.....	6,945,400	896,400	7,261,800

In which bank did the loans exceed the deposits? By how many dollars?

SUBTRACTION

9

15. Find the difference between the loans and deposits for each of the other banks.

16. Find the difference between the total loans and the short-time loans in the case of each bank.

17. From the following table find the increase in the number of post offices from 1865 to 1880. From 1865 to 1908.

FISCAL YEAR.	NUMBER OF POST OFFICES.	EXTENT OF POST ROUTES IN MILES.	REVENUE OF THE DEPARTMENT.	EXPENDITURE OF THE DEPARTMENT.
1865.....	20,550	142,340	\$14,556,159	\$13,694,728
1870.....	28,492	231,232	19,772,221	23,998,837
1875.....	35,547	277,873	26,791,360	33,611,309
1880.....	42,989	343,888	33,315,479	36,542,804
1885.....	51,252	365,251	42,560,844	49,533,150
1890.....	62,401	427,991	60,882,097	65,930,717
1895.....	70,064	456,026	76,983,128	86,790,172
1896.....	70,360	463,313	82,499,208	90,626,296
1897.....	71,022	470,032	82,665,462	94,077,242
1898.....	73,570	480,462	89,012,618	98,033,523
1899.....	75,000	496,948	95,021,384	101,632,160
1900.....	76,688	500,982	102,354,579	107,740,268
1901.....	76,945	511,808	111,631,193	115,554,920
1902.....	76,215	507,540	121,848,047	124,785,697
1903.....	74,169	506,268	134,224,443	138,784,488
1904.....	71,131	406,818	143,582,624	152,362,117
1905.....	68,131	486,805	152,826,585	167,399,169
1906.....	65,600	478,711	167,932,782	178,449,778
1907.....	62,663	463,406	183,585,005	190,238,288
1908.....	61,158	450,738	191,478,662	208,351,886

18. Make and solve five similar problems.

19. Make and solve similar problems about the extent of post routes.

20. Make and solve problems about the amounts of revenue. Also about the differences between revenue and expenditure.

MULTIPLICATION

11. Multiplication is a short method of finding the result of repeating a number as an addend.

EXAMPLE: What is the cost of 825 blankets at \$6 each? The solution is evidently $825 \times \$6$.

$$\begin{array}{r} 6 \\ 825 \\ \underline{30} \\ 120 \\ 4800 \\ \underline{4950} \end{array}$$

Since the multiplication at the left is awkward, we use the plan at the right.

We can do this because the product of two factors is the same no matter which factor is taken as the multiplier.

$$\begin{array}{r} 825 \\ \underline{6} \\ 4950 \end{array}$$

1. In the example below, which number is the multiplicand? The multiplier? The product?

$$\begin{array}{r} 8.421 \\ \underline{7.63} \\ .03 \times 8.421 - \quad .25263 \\ .6 \times 8.421 - \quad 5.0526 \\ 7 \times 8.421 - \quad 58.947 \\ \hline 64.25223 \end{array}$$

12. *The decimal point in the product is located by setting off from the right as many decimal places as there are in both multiplicand and multiplier.*

The work of multiplication should be tested by repeating the calculation.

WRITTEN EXERCISES

1. What is the cost of 963 chairs at \$5 each?
2. Find the cost of 1,045 stands at \$11 each.
3. Find the cost of 964 carpet sweepers at \$3 each.
4. Find the cost of 733 rugs at \$4 each.

5. Find the numbers that would fill out the last column in this table:

NUMBER.	BOOKS.	PRICE (EACH).	TOTAL COST.
500	Spellers	\$.15	()
250	Readers	.35	()
365	Classics	.60	()
783	Latin Grammars	1.10	()
1260	Dictionaries	3.50	()
876	Geographies	1.25	()
253	Physics	1.30	()
765	Geometries	1.25	()
980	Algebras	1.12	()
1343	Chemistries	1.18	()
25500	Primers	.45	()
54200	Grammars	.75	()

Multiply:

- | | | |
|------------------|-------------------|------------------|
| 6. 6.30 by 86. | 11. 724 by 86. | 16. 402 by 4.2. |
| 7. 1280 by 17. | 12. 8.46 by 1.33. | 17. 87.5 by .29. |
| 8. 15.07 by 29. | 13. 943 by 6.09. | 18. 300 by 7.6. |
| 9. 46.3 by 10.8. | 14. 712 by 50.8. | 19. 41.8 by .27. |
| 10. 19.7 by 999. | 15. 8.63 by 8.77. | 20. 609 by 4.1. |

Short Processes of Multiplication

ORAL EXERCISES

- How is an integer multiplied by 10? By 100? By 1000?
- How is a decimal multiplied by 10? By 100? By 1000?
- What part of 10 is 5? What is a short way to multiply a whole number by 5?
- What part of 100 is 25? What is a short way to multiply a whole number by 25?

5. 75 is what part of 100? If two ciphers are annexed to a whole number, and $\frac{3}{4}$ of the result is taken, by what is the number multiplied?

6. 125 is what part of 1000? How may a whole number be multiplied easily by 125? Illustrate the method. How may a whole number be multiplied easily by 1.25?

7. Multiply by 10: 18; 180; 1,800; 63; 630; 876; 806.

8. Multiply by 100: 6; 60; 600; 13; 130; 106; 66; 660.

9. Multiply by 1000: 3; 4; 5; 50; 23; 20; 145; 1,635.

10. Multiply by 10: .3; 9.4; 3.05; 16.67; 1.009; 90.09.

11. Multiply by 100: .06; .005; 6.05; 12.005; 12.635.

12. Multiply by 1000: .008; 2.005; 2.63; 46.73; 965.5.

13. Multiply by 5 without writing the figures:

36; 18; 28; 49; 70; 136; 43; 476.

14. Multiply by 25 without writing the figures:

16; 17; 24; 40; 13; 19; 116.

15. Multiply by 75 without writing the figures:

12; 16; 40; 24; 20; 28; 56.

Find the cost of:

16. 16 lb. of coffee at 25 cts. a pound.

17. 36 cans of baking powder at 25 cts. a can.

18. 84 cans of corn at 5 cts. each.

19. 124 bottles of olives at 25 cts. a bottle.

20. 82 doz. oranges at 50 cts. a dozen.

21. 24 cans of cherries at 25 cts. a can.

22. 136 Arithmetics at 45 cts. each.

23. 48 chairs at \$1.25 each.

24. 96 caps at 75 cts. each.

25. 112 doz. teaspoons at \$1.25 a dozen.

26. 320 pairs of skates at \$1.25 a pair.

27. 444 pairs of slippers at 75 cts. a pair.

13. $9 = 10$ minus what number? How many times a number must be taken from 10 times the number to leave 9 times the number? State a short way to multiply a number by 9.

$$\text{Thus, } 9 \times 181 = \begin{cases} 1810 \\ \underline{181} \\ 1629 \end{cases}$$

$99 = 100$ minus what number? How many times a number must be taken from 100 times the number to leave 99 times the number? State a short way to multiply by 99.

WRITTEN EXERCISES

1. Multiply by 9: 23; 54; 76; 97.
2. Find the cost of 47 packages of macaroni at 9 cts. each.
3. Find the cost of 381 packages of oatmeal at 9 cts. each.
4. Multiply by 99: 14; 27; 59; 84.
5. Find the cost of 52 shirt-waists at 99 cts. each.
6. Find the cost of 16 pairs of curtains at 99 cts. a pair.
Suggestion : $99 = 100$ minus what number?
7. Multiply by 999: 4; 51; 84; 26; 718.
8. Multiply by 98: 7; 15; 36; 49; 72.
Suggestion : $98 = 100$ minus what number?
9. Find the cost of 12 horses at \$98 each.
10. Multiply by 199: 3; 9; 52; 69; 81; 46.
11. Find the cost of 42 bbl. of flour at \$4.99 a barrel.
12. Multiply by 198: 6; 12; 93; 47.
13. Find the cost of 7 rugs at \$1.98 each.
14. Multiply by 298: 3; 16; 39; 57.
15. Find the cost of 16 doz. handkerchiefs at \$2.98 a dozen.
16. Find the cost of 17 bags of sugar at \$1.99 a bag.
17. Multiply by 399: 26; 52; 3; 17.
18. How much did a milliner receive for 24 hats at \$6.98?

14. When several factors are to be combined, the amount of work to be done sometimes depends upon the order in which the factors are taken.

EXAMPLE: Multiply $5 \times 7 \times 6$.

Which of the following is the easiest to do without writing if we begin at the left and multiply by the second number and then by the third:

- (1) $5 \times 7 \times 6 - (\quad)$, or (2) $7 \times 6 \times 5 - (\quad)$,
or (3) $5 \times 6 \times 7 - (\quad)$?

ORAL EXERCISES

Multiply:

- | | | |
|--------------------------------------|-----------------------------|------------------------------|
| 1. $2 \times 9 \times 5$. | 4. $7 \times 4 \times 5$. | 7. $14 \times 4 \times 5$. |
| 2. $11 \times 18 \times 5$. | 5. $5 \times 28 \times 3$. | 8. $82 \times 2 \times 5$. |
| 3. $2 \times 17 \times 3 \times 5$. | 6. $8 \times 13 \times 5$. | 9. $25 \times 84 \times 8$. |

15. For rapid oral work, it is often easier to multiply successively by the factors of the multiplier.

EXAMPLE: Multiply 14×15 .

From left to right, which is the easier to do without writing:

- (1) $14 \times 15 - (\quad)$, or (2) $2 \times 15 \times 7 - (\quad)$?

ORAL EXERCISES

- | | | | |
|---------------------|---------------------|----------------------|----------------------|
| 1. 16×45 . | 4. 25×14 . | 7. 150×18 . | 10. 22×45 . |
| 2. 24×50 . | 5. 18×35 . | 8. 162×15 . | 11. 35×16 . |
| 3. 38×50 . | 6. 36×50 . | 9. 12×45 . | 12. 25×84 . |

Solve:

13. A teamster hauled 24 cu. ft. of earth at a load. How many cubic feet did he haul in 45 loads?

14. There are 12 pairs of shoes in a case. How many pairs are there in 25 cases?

15. Which is the easier to multiply, 15×18 or $5 \times 18 \times 3$?

16. EXAMPLE: What is the cost of 468 books at 15 cts. each?

Solution: (1) Since, to multiply by 15 is to multiply by $1\frac{1}{2}$ times 10,

(2) Then, $468 \times 15 = 468 \times 1\frac{1}{2} \times 10 = 7,020$.

What part of a number is added to the number to multiply it by $1\frac{1}{2}$? How is a number multiplied by 10?

ORAL EXERCISES

Tell why the following are true, with any number as multiplicand:

TO MULTIPLY BY:	ADD TO MULTIPLICAND:	MOVE DECIMAL POINT:
1. 15.	$\frac{1}{2}$ of itself.	1 place to the right.
2. $13\frac{1}{2}$.	$\frac{1}{2}$ " "	1 " " " "
3. $133\frac{1}{2}$.	$\frac{1}{2}$ " "	2 places " "
4. $112\frac{1}{2}$.	$\frac{1}{2}$ " "	2 " " " "
TO MULTIPLY BY:	SUBTRACT FROM MULTIPLICAND:	MOVE DECIMAL POINT:
5. $87\frac{1}{2}$.	$\frac{1}{2}$ of itself.	2 places to the right.
6. $66\frac{2}{3}$.	$\frac{1}{3}$ " "	2 " " " "

WRITTEN EXERCISES

What is the cost of:

- 96 yd. of silk at $\$1.12\frac{1}{2}$ per yard?
- 56 yd. of carpet at $87\frac{1}{2}$ cts. per yard?
- 72 bu. of potatoes at $66\frac{2}{3}$ cts. per bushel?
- 66 yd. of ribbon at $13\frac{1}{2}$ cts. per yard?
- 45 yd. of lace at 15 cts. per yard?
- 28 yd. of velvet at $\$1.33\frac{1}{3}$ per yard?
- 14 yd. of gingham at $13\frac{1}{2}$ cts. per yard?
- 64 screw drivers at $87\frac{1}{2}$ cts. each?
- 25 rolls of wall paper at $87\frac{1}{2}$ cts. per roll?

Powers

17. A **power** of a number is the result of taking the number two or more times as a factor.

Thus, 2×2 , or 4, is the second power of 2; 5×5 , or 25, is the second power of 5; $4 \times 4 \times 4$, or 64, is the third power of 4, and so on.

ORAL EXERCISES

1. What other name has the second power? The third power?
2. What power of 3 is $3 \times 3 \times 3 \times 3$, or 81?
3. What power of 5 is $5 \times 5 \times 5 \times 5 \times 5$, or 3,125?
4. What is the square, or second power, of 3? Of 5?
5. What is the cube, or third power, of 2? Of 3? Of 5?
6. What is the area of a square whose side is 4 in.?
7. What is the volume of a cube whose edge is 4 in.?
8. 100 is what power of 10? 1000 is what power of 10?

18. The product of two like factors, as 2×2 , 3×3 , 4×4 , 5×5 , is written for brevity 2^2 , 3^2 , 4^2 , 5^2 . Similarly, $2 \times 2 \times 2$, $3 \times 3 \times 3$, $4 \times 4 \times 4$, and so on, may be written 2^3 , 3^3 , 4^3 , and so on.

19. Powers of 10 larger than 1000 are expressed more simply by the use of exponents.

Thus, 10,000 is 10^4 ; 50,000 is 5×10^4 ; 85,000 is 85×10^3 .

20. The small figure written above is called an **exponent** and shows how often the number above which it stands is taken as factor in the product.

ORAL EXERCISES

1. What exponent indicates the square of a number?
2. What number is expressed by each of the following: 1^2 ; 2^2 ; 3^2 ; 4^2 ; 5^2 ; 6^2 ; 7^2 ; 8^2 ; 9^2 ; 10^2 ?

3. If s is the side of a square, what product represents the area of the square?

4. What exponent is used to indicate the cube of a number? Indicate the cube of: 5; 6; 7; 8; 10; 15.

5. If e is the edge of a cube, what product represents the volume of the cube?

6. What is the number expressed by: 1^3 ; 2^3 ; 3^3 ; 4^3 ; 5^3 ?

WRITTEN EXERCISES

1. What is the value of: $2^3 \times 3^2$? 5×4^3 ? $3^2 \times 10^3$?

2. Write the factors of each of the following numbers, and express any powers by the use of exponents: 25; 50; 75; 18; 27; 63; 72; 90.

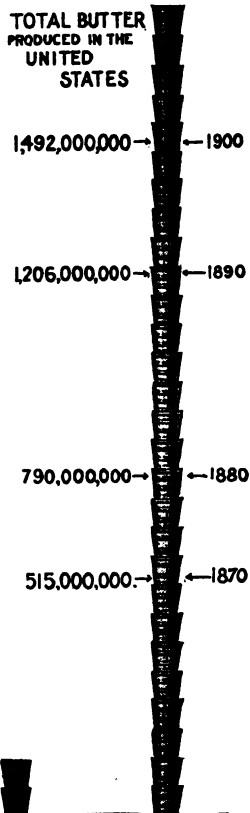
3. $600 = 6 \times 10 \times 10$. Write this by use of an exponent.

4. Write similarly by use of exponents: 300; 900; 1000; 5,000.

5. Write 300,000, using an exponent to express the power of 10. Also 900,000; 950,000; 895,000; 1,000,000; 9,000,000; 129,000,000; 1,236,000,000.

6. The diagram shows the number of million pounds of butter produced in the United States in 1870, 1880, 1890, and 1900. Read the amount produced in 1870; in 1880; in 1890; in 1900.

7. Using exponents, write all of the numbers in the diagram expressing amounts of butter; also that relating to oleomargarine.



DIVISION

21. Division is the inverse of multiplication. It is the process by which, if the product and one factor are given, we find the other factor.

22. In every division the divisor multiplied by the quotient and increased by the remainder, if any, equals the dividend; or

$$\text{Divisor} \times \text{Quotient} + \text{Remainder} = \text{Dividend}.$$

Thus, in $47 \div 9$, the quotient is 9, and the remainder 2; and $47 = 9 \times 5 + 2$.

ORAL EXERCISES

Find the numbers to fill the blanks :

	1.	2.	3.	4.	5.	6.
Dividend:	28	46	72	()	()	40
Divisor:	4	()	12	2.5	6	8
Quotient:	()	23	()	8	9.5	()

	7.	8.	9.	10.	11.	12.	13.
Dividend:	46	52	()	()	96	128	256
Divisor:	8	()	6	7	()	()	16
Quotient:	()	4	18	12	12	25	()
Remainder:	()	0	0	4	0	3	()

WRITTEN EXERCISES

Perform the following divisions :

- | | | |
|------------------------|------------------------|-------------------------|
| 1. $17.55 \div .39$. | 7. $190.9 \div .83$. | 13. $1.909 \div 2.3$. |
| 2. $3.465 \div 6.3$. | 8. $284.9 \div 3.7$. | 14. $64.48 \div .31$. |
| 3. $2846 \div 24$. | 9. $96064 \div 32$. | 15. $14641 \div 11$. |
| 4. $1599 \div 39$. | 10. $6334 \div 31$. | 16. $28346 \div 29$. |
| 5. $3216.84 \div 16$. | 11. $7683 \div 29$. | 17. $186.75 \div 25$. |
| 6. $96048 \div 48$. | 12. $6.25 \div 1.25$. | 18. $70.185 \div 3.5$. |

Find the price of each to the nearest cent if :

- | | |
|-----------------------------|-----------------------------------|
| 19. 36 sheep cost \$107. | 27. 24 handkerchiefs cost \$1.92. |
| 20. 48 chairs cost \$60. | 28. 17 bbl. of flour cost \$99. |
| 21. 160 baskets cost \$157. | 29. 16 cans of corn cost \$1.24. |
| 22. 74 books cost \$27. | 30. 46 pairs of gloves cost \$51. |
| 23. 52 hats cost \$93. | 31. 28 ostrich plumes cost \$94. |
| 24. 23 watches cost \$195. | 32. 15 mandolins cost \$146.25. |
| 25. 83 vests cost \$290. | 33. 61 saws cost \$50. |
| 26. 35 lamps cost \$40. | 34. 77 planes cost \$86.15. |

Find the average cost per mile of each ship canal; test your work :

CANAL	LENGTH.	COST.
35. Suez	90 miles	\$100,000,000.
36. Russian	16	10,000,000.
37. Manchester	35.5	75,000,000.
38. Kaiser Wilhelm	61	40,000,000.
39. Elbe and Trave	41	6,000,000.

Solve :

40. A grocer has 56 salesmen; his total weekly payment for their services is \$728. Find the average weekly salary of each salesman.

41. The total consumption of wool, as shown in the table, for any year when divided by the consumption *per capita* gives the total population for the corresponding year. Find the total population of the United States for each year mentioned. Omit the decimal part of each result.

YEAR.	TOTAL CONSUMPTION.	CONSUMPTION OF WOOL PER CAPITA.
1840...	76,710,602	4.49
1850...	129,354,978	5.58
1860...	213,832,799	6.80
1870...	305,770,597	7.93
1880...	427,184,530	8.52
1890...	548,167,332	8.75
1900...	483,865,236	6.34

23. *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

EXAMPLE: In a recent year the pay-roll of the city of New York contained the names of 43,000 persons. The total amount of the pay-roll was about \$38,000,000. Find within one cent the average compensation of each employee.

Divide both dividend and divisor by 1000 to simplify the work.

The decimal point of the quotient will stand over the decimal point of the dividend, if the first figure of the quotient is placed over the last figure of the first partial dividend. Show that this plan is followed in the work at the right.

How may the result be tested?

$$\begin{array}{r}
 883.72 \\
 43 \overline{)38000.000} \\
 \underline{344} \\
 360 \\
 \underline{344} \\
 160 \\
 \underline{129} \\
 310 \\
 \underline{301} \\
 90 \\
 \underline{86}
 \end{array}$$

ORAL EXERCISES

State the quotient:

1. $8,000 \div 400$. 4. $10,000 \div 500$. 7. $840,000 \div 12,000$.
2. $3,500 \div 700$. 5. $30,000 \div 5,000$. 8. $2,040,000 \div 80,000$.
3. $4,800 \div 30$. 6. $24,400 \div 60$. 9. $68,000,000 \div 200,000$.

WRITTEN EXERCISE

Solve:

The table shows approximately the population and assessed value of property in New York City from 1870 to 1900:

YEAR.	POPULATION.	ASSESSED VALUE.
1870.....	1,500,000	1,300,000,000
1880.....	1,900,000	1,400,000,000
1890.....	2,500,000	2,100,000,000
1900.....	3,400,000	3,600,000,000

Find to the nearest cent the value of the property for each person (*per capita*).

Short Processes of Division**ORAL EXERCISES**

1. How is a number ending in zero divided by 10? In an integer where is the decimal point supposed to be?
2. How is a number ending in two zeros divided by 100?
3. How many places is the decimal point moved to the left in dividing a number by 10? In dividing by 100? In dividing by 1000?
4. State the quotient of each of the following numbers when divided by 10: 20; 70; 50; 130; 180; 1,900.
5. State the quotient when each of the following numbers is divided by 100: 200; 400; 700; 1,300; 19,000.
6. Divide by 10: 16; 130; 1.3; 12.5; 7.25.
7. Divide by 100: 160; 26.3; 7.60; 183.6; 1900.7.
8. Divide by 1000: 285.6; 1234.5; 10,000; 10.36.
9. Since we may divide by 20 by first dividing by 10 and then dividing the result by 2, state a short way of dividing by 20.

<i>Divide by 20:</i>	10. 63.4.	12. 1.75.	14. 5.6.
	11. 18.4.	13. 40.3.	15. .104.
<i>Divide by 30:</i>	16. 2.7.	18. 1.96.	20. 38.52.
	17. .69.	19. 30.3.	21. 27.06.
<i>Divide by 200:</i>	22. 64.8.	24. 126.	26. 7.26.
	23. 19.1.	25. 420.	27. .42.
<i>Divide by 600:</i>	28. 2.46.	30. 720.	32. 82.2.
	29. 954.	31. 660.	33. 17.4.

WRITTEN EXERCISES

Perform the following divisions to nearest .1:

1. $8,746 \div 30$.
2. $8,400 \div 7,300$.
3. $48,529 \div 700$.
4. $26,284 \div 140$.
5. $490,380 \div 2,600$.
6. $923,500 \div 480$.

24. Since $15 = \frac{3}{2} \times 10$, to divide by 15 is to multiply by $\frac{2}{3}$ and divide by 10. To multiply by $\frac{2}{3}$ is to subtract $\frac{1}{3}$ of the multiplicand.

EXAMPLE: How many sewing machines at \$15 each can be bought for \$285?

Solution: (1) $\frac{2}{3}$ of 285 = 95. (2) $285 - 95 = 190$.
(3) $190 \div 10 = 19$.

The solution is given merely to indicate the mode of thinking. The problems are to be solved without written calculation.

ORAL EXERCISES

Divide by 15: 1. 4.5. 3. 1.05. 5. 7.5.
2. 46.5. 4. 900. 6. 5.25.

7. What part of a number taken from it leaves $\frac{2}{3}$ of the number?

8. How many yards of silk at \$1.12 $\frac{1}{2}$ a yard can be bought for \$900?

Solution: (1) $1.12\frac{1}{2} - \frac{2}{3}$ of 1. (2) $900 \div \frac{2}{3} = 900 \times \frac{3}{2} = 800$.

Divide by 1.12 $\frac{1}{2}$: 9. 45. 11. 189.
10. 54. 12. 99.

13. Since $112\frac{1}{2} = 100 \times 1.12\frac{1}{2}$, how may we divide 6,300 by 112 $\frac{1}{2}$?

WRITTEN EXERCISES

Divide by 112 $\frac{1}{2}$: 1. 85,500. 3. 2,061.
2. 473,400. 4. 853,290.

Divide by 11.25: 5. 76,320. 7. 567.9.
6. 8,883. 8. 482,850.

Divide by 1,125: 9. 297,000. 11. 4,860,000.
10. 78,300. 12. 927,360.

Divide by 150: 13. 7,800. 15. 52,650.
14. 29,700. 16. 293,280.

ORAL EXERCISES

Tell why the following are true, with any number as dividend:

TO DIVIDE BY:	SUBTRACT FROM DIVIDEND:	MOVE DECIMAL POINT:
1. 15.	$\frac{1}{3}$ of itself.	1 place to the left.
2. $13\frac{1}{3}$.	$\frac{1}{3}$ " "	1 " " " "
3. $133\frac{1}{3}$.	$\frac{1}{3}$ " "	2 places to the left.
4. $112\frac{1}{3}$.	$\frac{1}{3}$ " "	2 " " " "
TO DIVIDE BY:	ADD TO DIVIDEND:	MOVE DECIMAL POINT:
5. $87\frac{1}{2}$.	$\frac{1}{2}$ of itself.	2 places to the left.
6. $66\frac{2}{3}$.	$\frac{1}{3}$ " "	2 " " " "

WRITTEN EXERCISES

Find how many of each article can be bought for the amount given:

1. Yards of carpet at $\$.87\frac{1}{2}$ per yard.....	\$ 70.00.
2. Atomizers at $33\frac{1}{3}$ cts. each.....	40.00.
3. Concert phonographs at $\$66\frac{2}{3}$	600.00.
4. Parlor phonographs at $\$12\frac{1}{2}$ each.....	350.00.
5. Teakettles at $\$1.33\frac{1}{3}$ each.....	60.00.
6. Bookcases at \$15 each.....	855.00.
7. Pairs of dumb-bells at $\$1.12\frac{1}{2}$ per pair...	20.25.
8. Tennis rackets at $\$1.33\frac{1}{3}$ each.....	24.00.
9. Tents at $\$13\frac{1}{3}$ each.....	120.00.
10. Dictionaries at \$15 each.....	135.00.
11. Chairs at $\$1.33\frac{1}{3}$ each.....	80.00.
12. Land at $\$66\frac{2}{3}$ an acre.....	10,000.00.
13. Wagons at $\$87\frac{1}{2}$ each.....	6,300.00.
14. Wood at $\$1.12\frac{1}{2}$ a cord.....	27.00.

FACTORS AND MULTIPLES

Divisibility

25. All integers divisible by 2 are called **even** numbers. All integers that are not even are called **odd**.

26. Various tests for divisibility are stated in the following table:

A number is divisible :

By 2, if it ends in 0, 2, 4, 6, 8.

By 3, if the sum of its digits is divisible by 3.

By 4, if the number represented by the two digits at the right is divisible by 4.

By 5, if it ends in 0 or 5.

By 6, if divisible by 2 and by 3.

By 8, if the number represented by the three digits at the right is divisible by 8.

By 9, if the sum of its digits is divisible by 9.

By 10, if it ends in 0.

By 12, if it is divisible by 3 and by 4.

For 7 and 11, actual division is the best test.

ORAL EXERCISES

Which of the following are divisible by 3? By 4?

1. 128. 3. 276. 5. 648. 7. 87,000.

2. 585. 4. 3,284. 6. 950. 8. 11,656.

Which of the following could be set in 3 equal rows? In 4?

9. 126 cabbage plants. 13. 1,706 tomato plants.

10. 872 fruit trees. 14. 1000 pansies.

11. 126 strawberry plants. 15. 2,103 onion sets.

12. 432 raspberry bushes. 16. 2,468 rose bushes.

17. Name some multiples of 5. Is 45 divisible by 5? Is 50?
73? 90? 99? 109? 160?

18. Which of the following numbers are divisible by 2? By 3? By 4? By 5?

16	21	48	35	52	17	64
128	400	536	279	1,436	3,687	1,904

19. Which of the numbers above are divisible by 6? By 12?

Prime Factors

27. An integer that is divisible only by itself and 1 is called a **prime number**. If divisible by any other number it is called a **composite number**.

28. A factor that is a prime number is called a **prime factor**.

Every composite number can be expressed as a product of prime factors.

The prime factors may be found by dividing the number by any prime factor, then by dividing the quotient similarly by any of its prime factors, and so on, until a prime quotient is reached. The original number is the product of all the divisors and the last quotient. Thus, $60 = 2 \times 2 \times 3 \times 5$.

WRITTEN EXERCISES

Find the prime factors of:

- | | | | |
|-----------|------------|------------|------------|
| 1. 46. | 9. 111. | 17. 37. | 25. 9,333. |
| 2. 108. | 10. 144. | 18. 276. | 26. 1,964. |
| 3. 125. | 11. 625. | 19. 605. | 27. 1,728. |
| 4. 38. | 12. 51. | 20. 1000. | 28. 1,600. |
| 5. 2,240. | 13. 1,896. | 21. 1,900. | 29. 1,908. |
| 6. 4,030. | 14. 777. | 22. 813. | 30. 1,452. |
| 7. 211. | 15. 1,008. | 23. 6,990. | 31. 4,771. |
| 8. 8,451. | 16. 1,916. | 24. 2,433. | 32. 7,007. |

33. Express all the numbers to 100 as products of prime factors. Make a list of the prime numbers below 100.

Common Factors

29. A factor or divisor common to two or more integers is called a **common factor** or **common divisor** of the integers.

Thus, 3 is a common divisor of 9, 15, 33, and 60.

30. The largest integer that will divide without a remainder two or more given integers is called the **greatest common divisor** of those integers.

Thus, 12 is the greatest common divisor of 24, 84, and 360 because no larger integer is a common divisor of them all.

31. *To find the greatest common divisor of two or more numbers, divide the numbers by their smallest common prime factor. Divide the quotients obtained by their smallest common prime factor, and repeat the process until the quotients have no common prime factor. The greatest common divisor is the product of the prime divisors thus found.*

The greatest common divisors of large numbers not easily factored are not required in elementary arithmetic.

EXAMPLE: Find the g. c. d. of 24, 84, and 360.

$$\begin{array}{r} 2 \overline{) 24 \quad 84 \quad 360} \\ 2 \overline{) 12 \quad 42 \quad 180} \\ 3 \overline{) 6 \quad 21 \quad 90} \\ \quad 2 \quad 7 \quad 30 \end{array}$$

The numbers are first divided by 2, the smallest common prime factor.

The quotients are then divided by 2, their smallest common prime factor.

The new quotients are then divided by 3, their smallest common prime factor.

The quotients thus obtained have no common factor. The g. c. d. is the product of the successive prime divisors, or $2 \times 2 \times 3 = 12$.

WRITTEN EXERCISES

Find the g. c. d. of:

1. 26, 52, 130.

2. 18, 108, 756.

3. 70, 490, 238.

4. 38, 95, 209.

5. 36, 60, 1728.

6. 45, 265, 85.

7. 144, 1728, 84.

8. 90, 990, 195.

9. 25, 175, 625.

10. 21, 105, 315.

11. 49, 700, 749.

12. 85, 1000, 585.

Common Multiples

32. A number that is divisible by two or more numbers is called a **common multiple** of those numbers. The smallest number that can be divided by each of them is called their **least common multiple** (l. c. m.).

12, 24, and 36 are common multiples of 4, 6, and 12; and 12 is the least common multiple.

33. *To find the least common multiple of two or more numbers, factor the given numbers; multiply all of their prime factors, taking each one the greatest number of times that it occurs in any of the given numbers.*

EXAMPLES:

1. Find the l. c. m. of 28 and 42.

$28 = 2 \times 2 \times 7$, and $42 = 2 \times 3 \times 7$. The l. c. m. of 28 and 42 is $2 \times 2 \times 3 \times 7 = 84$. The l. c. m. contains 2 twice, because it is found in 28 twice. It contains 3 and 7 each once, because they occur once only in one or both of the given numbers.

2. Find the l. c. m. of 24, 56, 108.

$24 = 2 \times 2 \times 2 \times 3$.	The smallest number divisible by
$168 = 2 \times 2 \times 2 \times 3 \times 7$.	24 must contain the factors
$108 = 2 \times 2 \times 3 \times 3 \times 3$.	$2^3 \times 3$. To be divisible by 168
	it must also contain the factor 7. To be divisible by 108

it must contain, in addition to the above, the factor 3^2 . Therefore the l. c. m. of 24, 168, and 108 is $2^3 \times 3 \times 7 \times 3^2 = 2^3 \times 3^3 \times 7$, or 1512.

WRITTEN EXERCISES

Find the l. c. m. of:

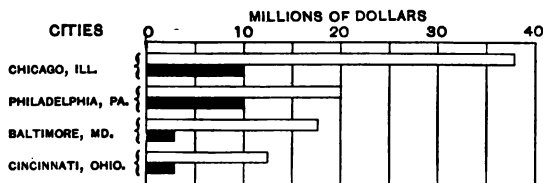
- | | | |
|-----------------|-------------------|--------------------|
| 1. 24, 56. | 5. 18, 15, 27. | 9. 256, 196, 180. |
| 2. 36, 56, 96. | 6. 55, 165, 121. | 10. 25, 125, 625. |
| 3. 22, 121, 77. | 7. 39, 169, 85. | 11. 360, 120, 90. |
| 4. 46, 18, 108. | 8. 12, 144, 1728. | 12. 48, 144, 1728. |

GRAPHICAL PROBLEMS

34. Number pictures, or graphs, enable us to show relations between given numbers.

ORAL EXERCISES

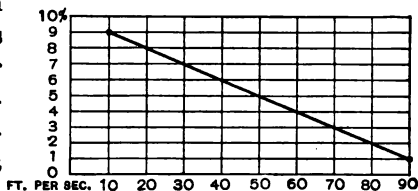
1. In the diagram the white strips represent the number



of million dollars' worth of men's clothing recently manufactured during one year in the cities named;

the black strips represent similarly the value of women's clothing. What was the value of Chicago's product of each kind? Of Philadelphia's? Of Baltimore's? Of Cincinnati's? Of Chicago's and Philadelphia's together?

2. When water flows through a ditch, some of it soaks into the soil. For soil of a certain character the diagram shows what per cent of the water entering the ditch soaks into the soil during the passage through the ditch at different rates of flow.



For example, when 10 cu. ft. of water enter during a second, 9% of it soaks into the soil.

What per cent is lost when 20 cu. ft. enter per second? 40 cu. ft.? 80 cu. ft.? 100 cu. ft.?

3. When 10 cu. ft. enter per second, how many cubic feet soak into the soil? How many cubic feet are delivered at the other end of the ditch per second? Per minute?

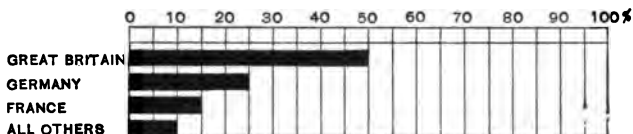
4. Make and solve 5 similar problems.

WRITTEN EXERCISES

1. Represent by a diagram as explained on page 28 :

CITY.	MEN'S CLOTHING.	WOMEN'S CLOTHING.
New York City.....	\$110,000,000	\$105,000,000
Boston.....	15,000,000	5,000,000
St. Louis.....	10,000,000	5,000,000

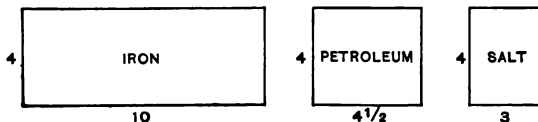
2. The following diagram shows by percentages how the 6,450,000 bales of cotton exported by the United States in one year were distributed:



How much was shipped to Great Britain? To France? To Germany? To all other places? The amount exported to Germany was what part of the amount shipped to Great Britain? Make and answer three similar questions from the diagram.

3. Indicate graphically that all of the spindles spinning the cotton were divided thus: Great Britain, 47 million; the European Continent, 34 million; the United States, 22 million; and the East Indies, 5 million.

4. These rectangles represent for a recent year the world's production of the minerals named:



Compute the areas of these figures. Each number found is the number of million tons of the mineral named. What amount of each mineral was produced?

5. In a recent year the acreage of corn, wheat, and oats and the number of bushels grown in the United States were:

	ACRES.	BUSHELS.	BUSHELS PER ACRE.
Corn.....	83,328,000	2,105,103,000	()
Wheat.....	42,504,000	()	12.3
Oats.....	()	809,280,000	30.0

Find the numbers to fill the blanks in the table.

6. If 1 sq. in. represents 1000 acres, how long must a rectangle 2 ft. wide be to represent the acreage of corn? Of wheat? Of oats?

7. Find the average yield of potatoes per acre from the table for each group of states and for the whole country:

GROUP OF STATES.	ACRES.	TONS OF POTATOES.
New England.....	180,000	505,000
Middle.....	410,000	1,440,000
Southern.....	240,000	420,000
Central.....	1,710,000	2,710,000
Pacific.....	110,000	290,000

8. Using $\frac{1}{4}$ in. to represent 5,000 tons, draw on the black-board parallel lines to represent these yields.

9. Using 1 in. to represent a million pounds, draw parallel lines to represent the following table of statistics on the production of tobacco:

STATES.	POUNDS.	ACRES.
Virginia	73,000,000	110,000
North Carolina.....	70,000,000	120,000
Wisconsin.....	50,000,000	40,000
Tennessee.....	34,000,000	43,000
Connecticut.....	25,000,000	14,000

REVIEW

ORAL EXERCISES

1. What is meant by numeration? By notation? What is meant by Roman notation? What year is represented by MDLXV? By MCMX? By DCXXVI?

2. Name the four periods into which integers of 12 places may be separated for reading.

3. Name the two periods into which decimals of 6 places may be separated for reading.

Read:

4. In a recent year there were 2,315,297 telephones of all kinds in the United States, and 5,070,554,553 messages were sent over them.

5. In a recent year the capacity of electric power plants in the United States was 1,845,048 horse powers, which was .0243 horse power per inhabitant.

6. In nearly all European countries a system of measures known as the metric or international system is used. The fundamental unit of this system is the *meter* which is a little over 39 in. in length. More closely:

$$1 \text{ meter} = 1.093633 \text{ yd.}$$

$$1 \text{ yd.} = .914383 \text{ meters.}$$

The meter is approximately .0000001 of the distance from the Equator to the North Pole.

Solve:

7. State and illustrate a way of testing the work of addition. State another way.

8. State and illustrate a way of testing the work of subtraction.

9. State and illustrate two short processes in the work of multiplication.

Perform the following multiplications without writing the figures :

- | | | |
|-------------------------------|-----------------------|------------------------------|
| 10. 5×26 . | 15. 25×52 . | 20. 1000×2.03 . |
| 11. 10×498 . | 16. 75×56 . | 21. $100 \times .023$. |
| 12. 100×38 . | 17. 75×440 . | 22. $6 \times 27 \times 5$. |
| 13. $2 \times 7 \times 5$. | 18. 9×67 . | 23. $100 \times .008$. |
| 14. $25 \times 63 \times 4$. | 19. 50×82 . | 24. $100 \times .146$. |

25. State and illustrate two short processes in division.

26. How may the number of decimal places in a product be determined? In a quotient?

State the quotient :

- | | | |
|---------------------|----------------------|-----------------------|
| 27. $84 \div 10$. | 29. $650 \div 500$. | 31. $3,640 \div 40$. |
| 28. $740 \div 20$. | 30. $6.3 \div 10$. | 32. $9,000 \div 15$. |

33. State five tests of divisibility.

34. Name by letter the numbers of the following list that are divisible by 5:

- | | | | |
|---------|------------|------------|-------------|
| (a) 16. | (d) 4,000. | (g) 3,648. | (j) 43,785. |
| (b) 75. | (e) 5,640. | (h) 8,379. | (k) 29,263. |
| (c) 80. | (f) 2,715. | (i) 2,916. | (l) 70,856. |

35. Define greatest common divisor. Also, least common multiple.

36. What is an exponent? Illustrate the use of exponents.

37. State a principle in division that enables us to make the divisor a whole number before performing the division.

38. What is meant by "casting out nines"? What use can be made of this process?

39. Define: odd number; even number; prime number; composite number.

40. How is the greatest common divisor of two or more numbers found? How is their least common multiple found?

41. How is division tested?

WRITTEN EXERCISES

Add and test :

1. 48,563	2. 84,027	3. 291,829	4. 72.091
27,097	2,986	843,962	360.059
36,984	90,027	739,872	987.836
50,126	17,963	385,746	483.
73,295	84,137	597,835	2.007
14,872	39,562	458,953	879.43
<u>62,311</u>	<u>78,358</u>	<u>622,384</u>	<u>26.748</u>

Find the cost of :

5. 72 books at \$1.12 $\frac{1}{2}$ each.
6. 582 yd. calico at 13 $\frac{1}{2}$ cts. per yard.
7. 624 planes at 87 $\frac{1}{2}$ cts. each.
8. 32 doz. pr. slippers at \$16 $\frac{1}{2}$ per dozen.

Solve :

9. In a recent year, the total revenues of the commercial telegraphic systems of the United States were: From telegraph traffic, \$35,300,569; from other sources, \$5,629,469. The total expenses were: Salaries and wages, \$15,039,673; operation and maintenance, \$9,220,948; interest, \$1,950,282; dividends, \$6,-256,693; other expenses, \$4,737,131. Find the excess of the total revenues over the total expenses.

10. In a recent year 91,655,287 telegraphic messages were sent in the United States at a total cost of \$29,118,089. Find the average cost per message to the nearest .1 cent.

11. In a recent year, the telephone companies of the United States employed 14,124 salaried officials, clerks, etc., at a total annual salary of \$9,885,886. Find the average annual salary to nearest dollar.

12. The same companies employed 64,628 wage earners with total wages of \$26,369,735. Find the average annual wages of each employee.

Write by use of exponents:

13. $3 \times 3 \times 2$.

17. $2 \times 5 \times 5 \times 5$.

14. $3 \times 3 \times 2 \times 2$.

18. $10 \times 10 \times 2 \times 2 \times 2$.

15. $5 \times 5 \times 10 \times 10$.

19. $10 \times 10 \times 10 \times 10 \times 10$.

16. $3 \times 3 \times 3 \times 3$.

20. $3 \times 2 \times 5 \times 2 \times 3$.

21. Write as a product of factors, using exponents where applicable: 50; 200; 1000; 5,000; 8,000.

22. 9,945 tons of iron ore were unloaded from a steamer in $4\frac{1}{2}$ hr. How many tons were unloaded per hour?

23. A large cattle ranch in Texas contains 700,000 acres. What is the land worth at \$7.85 an acre?

24. The table shows the number and value of the railway cars manufactured in the United States in a recent year:

KIND OF CAR.	NUMBER.	TOTAL VALUE.
Passenger coach.....	181	\$957,526
Sleeping car.....	194	2,767,061
Dining car.....	37	404,503
Freight car.....	116,590	62,161,013

Find the average value of a car of each kind (to nearest \$100).

25. In Exercise 24 find the average value to the nearest cent.

26. The woodland of this country is about $\frac{1}{3}$ of the total area, 3,025,600 sq. mi. How many square miles are woodland?

27. How many acres of woodland are there in the United States? (640 acres = 1 sq. mi.)

28. The Government and many of the states have established forest preserves as follows:

1891.....	13,457,680 acres	1898-1901....	7,050,089 acres
1893.....	4,443,000 acres	1902-1904....	15,995,196 acres

Find the total acreage of these preserves. How many more acres were established between 1902 and 1904 than in 1891? How many more between 1898 and 1901 than in 1893?

29. In 1900 there were 33,035 establishments manufacturing from lumber \$611,611,524 worth of products. What was the average output of each factory?

30. These factories employed 283,260 workmen at an average wage of \$370 a year. Find their total earnings.

31. In 1900 the lumber mills sawed 35,084,166,000 board feet, valued at \$566,832,984; this was an increase of $\frac{1}{4}$ over the value of the product in 1890. What was that value?

32. It is estimated that trees containing 21,239,000,000 ft. of spruce are now standing in Maine and increasing at the rate of 600 million feet a year. If none are destroyed, how many feet will there be in 25 yr.?

33. Taking the annual capacity of the Maine pulp mills to be 275 million feet, how much may be used for other purposes without exceeding the natural increase of the timber?

34. A farmer kept the following account of a field of corn:

EXPENSES PER ACRE.		RETURNS PER ACRE.
Plowing..... \$2.50	Seeding..... \$1.65	50 bu. at.... \$.45
Harrowing..... 2.50	Cultivating... 4.50	Fodder..... 8.00
Fertilizing..... .80	Harvesting... 8.00	

Find the total returns, expenses, and net profit per acre. What was the net profit from 25 acres of corn?

35. The street railways of New York City carried, in all, 1,036,000,000 passengers in a recent year. Taking the population as 3,400,000, what was the average number of times that each inhabitant used the street railways?

36. What is the cost of 7 yd. of cloth at \$1.33 $\frac{1}{3}$ a yard?

37. A Washington lumber mill cuts 87 thousand feet of lumber every 10 hr. How many feet will it cut in 26 days of 10 hr. each? If an ordinary 10-room house requires 7 thousand feet of lumber, how many such houses will one day's product build?

38. Some orange groves net their owners \$875 an acre. At this rate what is the profit on $55\frac{1}{2}$ acres?

39. The value of the furniture manufactured annually in this country is \$80,000,000. There are about 1,500 factories. What is the average output for each factory?

40. The large wheat farms of Minnesota and the Dakotas contain from 3,000 to 10,000 acres of land. How many bushels of wheat would a farmer harvest from 4,625 acres yielding 15 bu. an acre?

41. What would this wheat bring at 60 cts. a bushel?

42. The grain was threshed in the field by 10 threshing machines, each having a capacity of 400 bu. a day. How many days did it take to thresh the wheat?

43. The grain was loaded on cars carrying 900 bu. each. How many cars were required for the crop?

44. $4\frac{1}{2}$ bu. of wheat are used to make a barrel of flour. How many barrels would this crop make?

45. A ranch had 100,000 cattle; its owner employed 300 cowboys. How many was this for every 1000 cattle?

46. For grazing, cattle are grouped in herds of from 125 to 200 each. How many herds of 125 each are there in 100,000 cattle?

47. What would 1,200 ponies for herding the cattle cost at \$29.75 each?

48. How many cattle cars carrying 19 steers each are needed to carry 6,099 steers?

49. 6,099 steers averaged 800 lb. in weight and sold at 5 cts. a pound. How many dollars was this?

50. On a ranch in Kansas a blizzard froze $\frac{1}{4}$ of a herd of 15,000 cattle. How many were frozen?

51. About 20 million dollars' worth of beef is exported annually; of this the British Isles take .99. What is the value of the beef exported to all the rest of the world?

52. Find the cost of growing 5.5 acres of beets when the cost per acre of preparing the soil is \$1.55; of seed and planting, \$3.25; and of cultivating and care, \$31.

53. If it costs \$230.20 to grow and harvest a 5.5-acre crop of beets, what is the cost of harvesting alone?

54. If the beets from 1 acre sell for \$56.04 at \$4.38 a ton, what is the yield from each acre?

55. At this rate how many tons were grown on the 5.5 acres? What was the total profit? What was the profit on each acre?

56. If the total receipts for sugar, molasses, and pulp is \$11.10 per ton of beets, and if the total cost of these products is \$7.84 per ton, what is the profit to the manufacturer?

57. Represent graphically the coal production of the following countries in a recent year:

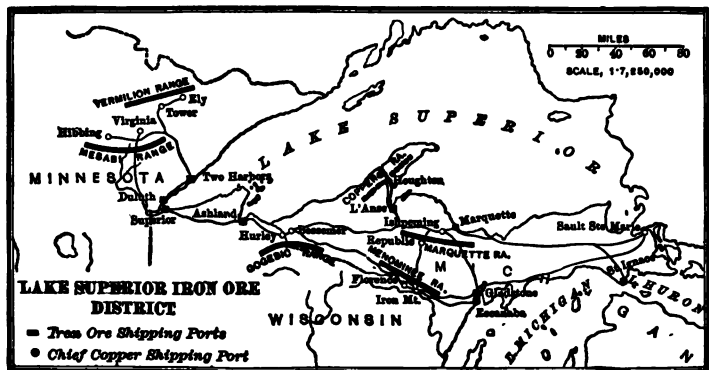
COUNTRY.	MILLION TONS.	COUNTRY.	MILLION TONS.
United States..	266	France.....	32
British Isles....	223	Belgium.....	22
Germany.....	153	Russia.....	16
Austria.....	40	All others.....	25

58. Taking the total of Exercise 57 to represent the world's annual consumption and 650,000,000,000 to be the total deposits, how many years would it take to exhaust the supply?

59. Taking the average production of honey per colony of bees to be 35 lb. a season and the selling price of the honey to be 9 cts., what is the value of the product of an apiary of 200 colonies? If it takes $\frac{1}{3}$ of this amount to meet incidental expenses and to pay the interest on the investment, what is the profit of the apiary for one season?

60. The area over which a colony of bees generally ranges is equal to a square 5 mi. on a side. How many acres is this?

61. A good bee pasture should average 1 honey-bearing plant per square foot. How many honey-bearing plants are there in 25 sq. mi. of such pasture?



62. The total production of iron ore in the Lake Superior district has reached 12,000,000 tons in one year. What was the average production for each of the five "ranges"?

63. Much of the ore is taken with steam shovels, averaging 1,800 tons a day. How many days would it take 25 such shovels to load 8,000,000 tons of ore?

64. What was the cost of mining this output at $23\frac{1}{4}$ cts. a ton?

65. The other 4,000,000 tons were mined underground. What was the cost of mining at $89\frac{1}{2}$ cts. a ton?

66. 3,000,000 tons have been shipped in one year from the ten ports of Lake Michigan and Lake Superior. Find the average tonnage per port.

67. The iron and steel produced in the United States in a recent year was 33 million tons. This was about $\frac{1}{11}$ of the amount produced in all countries. How many million tons were there in the world's production?

68. The amount of iron used by the people of the United States averages 300 lb. *per capita*. What was the consumption in 1900 when the population was 76 million?

CHAPTER II

FRACTIONS

REDUCTION

35. One or more of the equal parts of a quantity, thought of as a unit, is called a **fraction**.

Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$ are fractions.

36. Any one of the equal parts of a unit, or thing, is called a **fractional unit**.

Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{10}$ are fractional units.

ORAL EXERCISES

1. How may $\frac{2}{3}$ be changed to $\frac{4}{8}$? $\frac{1}{2} = \frac{?}{8}$? $\frac{3}{4} = \frac{?}{8}$?

2. What effect is produced upon the value of a fraction by multiplying or dividing both terms by the same number?

3. How may a fraction be changed to an equal fraction with a different denominator?

4. $\frac{1}{2} = \frac{?}{8}$?

6. $\frac{1}{2} = \frac{?}{8}$?

8. $\frac{1}{2} = \frac{?}{10}$?

10. $\frac{2}{12} = \frac{?}{4}$?

5. $\frac{1}{3} = \frac{?}{6}$?

7. $\frac{1}{3} = \frac{?}{6}$?

9. $\frac{1}{2} = \frac{?}{8}$?

11. $\frac{1}{2} = \frac{?}{8}$?

12. 3 lb. and 4 oz. are how many ounces? In what unit are they expressed before adding?

13. How must numbers be expressed before they can be added or subtracted?

14. What is the fractional unit in each of the following fractions: $\frac{2}{3}$? $\frac{4}{7}$? $\frac{9}{11}$? $\frac{13}{14}$? $\frac{20}{100}$? $\frac{1}{4}$? $\frac{3}{8}$? $\frac{4}{11}$?

15. Fractions expressed in the same fractional unit are alike in which term?

16. What change must be made in fractions with different denominators in order to make them fractions having the same fractional unit?

37. To reduce fractions to equal fractions having the least common denominator, find the l. c. m. of the given denominators and change each fraction to an equal fraction having the least common multiple as denominator.

Thus, in $\frac{2}{3}$, $\frac{11}{12}$, $\frac{7}{15}$, the least common multiple of the denominator is 60.

$$60 \div 3 = 20$$

$$60 \div 12 = 5$$

$$60 \div 15 = 4$$

$$\frac{2 \times 20}{3 \times 20} = \frac{40}{60}$$

$$\frac{5 \times 11}{12 \times 5} = \frac{55}{60}$$

$$\frac{4 \times 7}{15 \times 4} = \frac{28}{60}$$

WRITTEN EXERCISES

Find the least common denominator of:

1. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{10}$.

2. $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{3}$.

3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$.

Reduce each of the following sets of fractions to fractions having the least common denominator:

4. $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{4}$.

9. $\frac{1}{16}$, $\frac{3}{8}$, $\frac{1}{4}$.

14. $\frac{5}{8}$, $\frac{1}{2}$, $\frac{11}{16}$.

5. $\frac{2}{3}$, $\frac{5}{8}$, $\frac{9}{16}$.

10. $\frac{3}{20}$, $\frac{5}{44}$, $\frac{2}{11}$.

15. $\frac{5}{8}$, $\frac{7}{24}$, $\frac{2}{3}$.

6. $\frac{3}{40}$, $\frac{19}{80}$, $\frac{9}{160}$.

11. $\frac{5}{36}$, $\frac{7}{70}$, $\frac{13}{140}$.

16. $\frac{5}{12}$, $\frac{11}{16}$, $\frac{7}{24}$.

7. $\frac{11}{12}$, $\frac{7}{24}$, $\frac{5}{108}$.

12. $\frac{1}{40}$, $\frac{1}{80}$, $\frac{1}{32}$.

17. $\frac{19}{12}$, $\frac{2}{3}$, $\frac{5}{12}$.

8. $\frac{5}{12}$, $\frac{5}{144}$, $\frac{5}{1728}$.

13. $\frac{2}{40}$, $\frac{2}{800}$, $\frac{2}{10000}$.

18. $\frac{7}{108}$, $\frac{5}{84}$, $\frac{2}{21}$.

38. To reduce an improper fraction to a whole or a mixed number, divide the numerator by the denominator.

Thus, $\frac{24}{6} = 24 \div 6 = 4$. Also, $\frac{38}{5} = 38 \div 5 = 7\frac{3}{5}$.

39. To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator, and place the result over the given denominator.

Thus, $9\frac{8}{9} = \frac{8 \times 9 + 5}{9} = \frac{77}{9}$.

WRITTEN EXERCISES

Reduce to a whole or a mixed number :

- | | | | | |
|----------------------|------------------------|------------------------|------------------------|------------------------|
| 1. $1\frac{3}{8}$. | 6. $3\frac{2}{3}$. | 11. $4\frac{4}{7}$. | 16. $8\frac{40}{9}$. | 21. $8\frac{24}{7}$. |
| 2. $2\frac{6}{8}$. | 7. $1\frac{2}{7}$. | 12. $9\frac{3}{3}$. | 17. $7\frac{3}{3}$. | 22. $5\frac{24}{3}$. |
| 3. $8\frac{5}{13}$. | 8. $6\frac{3}{14}$. | 13. $12\frac{1}{20}$. | 18. $2\frac{01}{16}$. | 23. $4\frac{3}{16}$. |
| 4. $7\frac{6}{8}$. | 9. $1\frac{00}{9}$. | 14. $2\frac{63}{8}$. | 19. $7\frac{2}{40}$. | 24. $2\frac{60}{8}$. |
| 5. $6\frac{3}{10}$. | 10. $12\frac{3}{30}$. | 15. $3\frac{3}{9}$. | 20. $5\frac{27}{30}$. | 25. $2\frac{48}{60}$. |

Reduce to an improper fraction :

- | | | | | |
|-----------------------|-----------------------|-----------------------|------------------------|------------------------|
| 26. $5\frac{3}{8}$. | 31. $73\frac{3}{8}$. | 36. $18\frac{3}{8}$. | 41. $46\frac{9}{10}$. | 46. $72\frac{9}{10}$. |
| 27. $15\frac{3}{4}$. | 32. $19\frac{3}{4}$. | 37. $45\frac{3}{4}$. | 42. $7\frac{1}{2}$. | 47. $86\frac{1}{2}$. |
| 28. $26\frac{1}{2}$. | 33. $41\frac{1}{2}$. | 38. $19\frac{1}{2}$. | 43. $8\frac{3}{8}$. | 48. $12\frac{3}{8}$. |
| 29. $4\frac{5}{8}$. | 34. $16\frac{9}{8}$. | 39. $5\frac{4}{8}$. | 44. $11\frac{5}{8}$. | 49. $16\frac{1}{2}$. |
| 30. $8\frac{3}{8}$. | 35. $72\frac{3}{4}$. | 40. $8\frac{1}{2}$. | 45. $29\frac{3}{8}$. | 50. $32\frac{1}{2}$. |

ADDITION AND SUBTRACTION

40. *Fractions having the same denominators are added or subtracted by adding or subtracting their numerators and placing the result over the common denominator. If the fractions have unequal denominators, reduce them to fractions having the l. c. d.*

WRITTEN EXERCISES

Add or subtract the following fractions as indicated :

- | | | |
|-----------------------------------|-------------------------------------|-------------------------------------|
| 1. $\frac{3}{11} + \frac{7}{8}$. | 8. $\frac{9}{16} + \frac{11}{32}$. | 15. $\frac{4}{9} + \frac{8}{45}$. |
| 2. $1\frac{2}{3} + \frac{3}{4}$. | 9. $\frac{1}{2} + \frac{9}{16}$. | 16. $\frac{7}{10} + \frac{1}{15}$. |
| 3. $\frac{9}{4} + \frac{7}{8}$. | 10. $\frac{5}{8} - \frac{1}{2}$. | 17. $2\frac{5}{7} - \frac{5}{36}$. |
| 4. $1\frac{2}{7} - \frac{9}{4}$. | 11. $\frac{1}{2} - \frac{9}{16}$. | 18. $\frac{4}{9} - \frac{8}{45}$. |
| 5. $\frac{3}{11} - \frac{7}{8}$. | 12. $\frac{1}{2} - \frac{3}{16}$. | 19. $\frac{7}{16} - \frac{7}{64}$. |
| 6. $\frac{5}{8} + 1\frac{1}{2}$. | 13. $\frac{5}{24} + \frac{9}{30}$. | 20. $\frac{4}{7} - \frac{3}{14}$. |
| 7. $\frac{2}{3} - \frac{1}{8}$. | 14. $\frac{7}{18} - \frac{3}{8}$. | 21. $\frac{5}{36} + \frac{11}{8}$. |

22. $\frac{1}{2} - \frac{2}{4} - \frac{1}{4}$.

26. $\frac{3}{4} - \frac{2}{8} + \frac{9}{10}$.

30. $\frac{5}{8} - \frac{1}{4} + \frac{7}{12}$.

23. $\frac{1}{2} + \frac{7}{8} - \frac{9}{16}$.

27. $\frac{5}{8} + \frac{3}{16} + \frac{1}{4}$.

31. $\frac{25}{4} - \frac{3}{2} + \frac{5}{18}$.

24. $\frac{5}{4} + \frac{25}{48} - \frac{8}{3}$.

28. $\frac{7}{12} + \frac{5}{6} - \frac{1}{3}$.

32. $\frac{11}{6} + \frac{7}{30} - \frac{9}{10}$.

25. $\frac{1}{2} + \frac{5}{8} - \frac{5}{16}$.

29. $\frac{7}{20} - \frac{3}{40} - \frac{7}{60}$.

33. $\frac{7}{10} + \frac{3}{20} - \frac{3}{25}$.

34. $\frac{5}{8} - \frac{7}{16} + \frac{11}{32}$.

35. $\frac{3}{25} + \frac{1}{4} + \frac{7}{10}$.

36. $\frac{3}{4} + \frac{1}{3} + \frac{5}{12} - \frac{2}{3} + \frac{5}{6} - \frac{7}{8} + \frac{4}{3} - \frac{3}{8} + \frac{7}{12} + \frac{8}{9} - \frac{3}{16} - \frac{1}{12}$.

37. $\frac{1}{8} + \frac{5}{16} + \frac{3}{32} - \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{9}{16}$.

38. $\frac{2}{3} + \frac{1}{6} - \frac{5}{12} + \frac{4}{3} - \frac{9}{16} + \frac{2}{30}$.

Solve :

39. A banker owns $\frac{1}{8}$ of the stock of a certain bank, another banker owns $\frac{3}{8}$, and still another owns $\frac{1}{4}$. What part of the whole is not owned by these three men together?

40. A farm is $\frac{3}{4}$ of a mile in length on one side, $\frac{1}{3}$ of a mile on another side, $\frac{5}{6}$ of a mile on the third side, and $\frac{11}{12}$ of a mile on the fourth side. How far is it around the farm?

41. A teamster hauled away $\frac{4}{12}$ of a pile of sand in the forenoon, and $\frac{4}{12}$ of the pile in the afternoon. What part of the pile remained at evening?

41. A number consisting of an integer and a fraction is called a **mixed number**.

42. *In adding or subtracting mixed numbers, the integers and fractions may be treated separately and the results combined, or the mixed numbers may first be reduced to improper fractions and then added or subtracted.*

EXAMPLES :

1. Add $17\frac{2}{3}$, $5\frac{1}{3}$, 10, $8\frac{1}{2}$.

Solution :

$$17\frac{2}{3} = 17\frac{4}{6}$$

$$5\frac{1}{3} = 5\frac{2}{6}$$

$$10 = 10$$

$$8\frac{1}{2} = 8\frac{3}{6}$$

$$\text{Sum} = 40\frac{9}{6} = 41\frac{7}{6}$$

If the fractions are expressed in different fractional units, what change must be made in the fractions? How is the work completed?

2. Subtract $5\frac{3}{4}$ from $17\frac{1}{4}$.

Solution :

$$\begin{array}{r} 17\frac{1}{4} = 16\frac{4}{4} \\ 5\frac{3}{4} = 5\frac{3}{4} \\ \hline \text{Diff.} = 11\frac{1}{4} = 11\frac{1}{4}. \end{array}$$

If the fraction in the minuend is less than the fraction in the subtrahend, what change must be made? Subtract the integers and fractions separately. How are the results combined?

WRITTEN EXERCISES

1. To what kind of fraction may a mixed number be changed? Change $19\frac{3}{8}$ to a fraction.

2. Change $7\frac{5}{8}$ and $15\frac{3}{4}$ to improper fractions and find their sum.

Add or subtract as indicated :

- | | | |
|--|---|--|
| 3. $\frac{9}{18} + 1\frac{1}{8}$. | 13. $\frac{7}{8} + 3\frac{3}{8}$. | 23. $\frac{9}{32} + 2\frac{7}{16}$. |
| 4. $1\frac{1}{8} - \frac{9}{16}$. | 14. $3\frac{3}{8} - \frac{7}{8}$. | 24. $2\frac{7}{16} - \frac{9}{32}$. |
| 5. $\frac{8}{9} + \frac{7}{48} + \frac{1}{3}$. | 15. $1\frac{1}{2} + \frac{3}{4} + \frac{7}{16}$. | 25. $\frac{5}{8} + \frac{5}{24} + \frac{5}{6}$. |
| 6. $\frac{8}{9} - \frac{7}{48} - \frac{1}{3}$. | 16. $1\frac{1}{2} - \frac{3}{4} - \frac{1}{16}$. | 26. $\frac{25}{84} - \frac{5}{6} - \frac{5}{36}$. |
| 7. $40\frac{3}{4} - 17\frac{5}{12}$. | 17. $52\frac{5}{8} - 12\frac{3}{16}$. | 27. $75\frac{1}{2} - 1\frac{3}{4}$. |
| 8. $170\frac{5}{8} - 7\frac{7}{8}$. | 18. $84\frac{1}{8} - 7\frac{1}{8}$. | 28. $84\frac{1}{8} - 12\frac{5}{8}$. |
| 9. $40\frac{3}{4} + 17\frac{5}{12}$. | 19. $75\frac{1}{2} + 1\frac{3}{4}$. | 29. $84\frac{1}{8} + 7\frac{1}{8}$. |
| 10. $84\frac{1}{8} + 12\frac{5}{8}$. | 20. $7\frac{5}{8} + 120\frac{7}{16}$. | 30. $40\frac{3}{4} + 91\frac{3}{8}$. |
| 11. $6\frac{1}{4} + 3\frac{9}{4}$. | 21. $26\frac{3}{10} - 19\frac{7}{10}$. | 31. $23\frac{1}{10} - 17\frac{3}{10}$. |
| 12. $12\frac{1}{3} - 3\frac{4}{7}$. | 22. $5\frac{1}{4} + 8\frac{1}{2}$. | 32. $8\frac{2}{3} + 6\frac{1}{10}$. |
| 33. $20\frac{3}{8} + 119\frac{7}{8}$. | 41. $705\frac{1}{2} - 126\frac{3}{4} - \frac{3}{8}$. | |
| 34. $100\frac{9}{10} + 999\frac{7}{10}$. | 42. $12\frac{3}{4} + 18\frac{3}{4} + \frac{9}{8}$. | |
| 35. $146\frac{3}{8} - 123\frac{1}{8}$. | 43. $145\frac{3}{8} + 9\frac{1}{8} + 3\frac{1}{8}$. | |
| 36. $17\frac{1}{2} + \frac{5}{8} + 4\frac{5}{8}$. | 44. $40\frac{3}{8} + 1\frac{1}{2} + 9\frac{7}{10}$. | |
| 37. $827\frac{3}{4} - 659\frac{1}{4}$. | 45. $39\frac{3}{10} + 6\frac{1}{2} + 12\frac{1}{2}$. | |
| 38. $437\frac{5}{8} - 118\frac{7}{8}$. | 46. $600 - 11\frac{1}{2} - 18\frac{7}{10}$. | |
| 39. $700\frac{1}{10} - 635\frac{1}{10}$. | 47. $946 - 187\frac{7}{10} - 352\frac{7}{10}$. | |
| 40. $365\frac{1}{4} + 379\frac{5}{8}$. | 48. $48\frac{3}{8} - 16\frac{9}{10} + 4\frac{1}{2}$. | |

Solve :

49. From the sum of $5.86\frac{1}{2}$ and $1.45\frac{5}{8}$ take $2.93\frac{7}{8}$.

50. From a roll of cloth containing $17\frac{1}{2}$ yd., $9\frac{1}{2}$ yd. are cut. How many yards remain in the roll?

51. Two garments, one requiring $3\frac{1}{2}$ yd. and another $5\frac{3}{4}$ yd. are cut from a piece of cloth containing $10\frac{1}{2}$ yd. How many yards are left?

52. Five cars contained the following loads of coal:

$30\frac{1}{2}$ T. $40\frac{3}{10}$ T. $49\frac{1}{2}$ T. $36\frac{1}{2}$ T. $45\frac{1}{2}$ T.

Find the total weight of the 5 loads.

53. The prices of wheat for 6 days were:

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
$86\frac{1}{8}\phi$	$85\frac{3}{8}\phi$	$88\frac{3}{4}\phi$	$90\frac{5}{8}\phi$	89ϕ	$87\frac{1}{8}\phi$

On which day was the price the lowest? How much above the lowest price was the price for each day?

54. The rear wheel of a wagon was $4\frac{5}{8}$ ft. high, and the front wheel $3\frac{5}{4}$ ft. high. How much higher was the rear wheel than the front one?

55. The pedestal of a statue is $4\frac{7}{8}$ ft. high; the statue is $5\frac{1}{2}$ ft. high. How high are the two together? How much higher is the statue than the pedestal?

56. The four sides of an irregular lot were of the following lengths: $84\frac{1}{2}$ ft., $163\frac{3}{4}$ ft., $83\frac{1}{2}$ ft., $162\frac{1}{2}$ ft. What was the distance around the lot?

57. A tank contained 800 gal. of oil; $273\frac{7}{10}$ gal. were drawn out Monday, $169\frac{1}{2}$ gal. Tuesday, $78\frac{3}{4}$ gal. Wednesday. How many gallons were then left in the tank?

58. Three villages, A, B, C, are connected by a straight road; B lies $16\frac{3}{4}$ mi. east of A, and C $27\frac{3}{4}$ mi. east of B. How far is it from A to C?

59. If a traveler is $7\frac{2}{10}$ mi. east of B in the previous exercise, how far is he from A? From C?

MULTIPLICATION

43. *The product of two or more fractions is a fraction whose numerator is the product of the given numerators and whose denominator is the product of the given denominators.*

EXAMPLE: Multiply $\frac{7}{8}$ by $\frac{9}{11}$.

$$\frac{7}{8} \times \frac{9}{11} = \frac{7 \times 9}{8 \times 11} = \frac{63}{88}.$$

44. *To find the product of an integer and a fraction, write the integer with denominator 1 and proceed as with two fractions.*

EXAMPLE: Multiply 12 by $\frac{5}{8}$.

$$12 \times \frac{5}{8} = \frac{12}{1} \times \frac{5}{8} = \frac{60}{8} = 7\frac{1}{2}.$$

Since multiplying the denominator by 1 leaves it unaltered, we may multiply a fraction by an integer by multiplying the numerator by that integer.

45. *To multiply mixed numbers, first reduce them to improper fractions.*

EXAMPLE: $4\frac{1}{3} \times 5\frac{1}{4} = \frac{13}{3} \times \frac{21}{4} = \frac{273}{12}.$

If only one factor is a mixed number, it may be easier to multiply directly. For example, multiply 196 by $15\frac{3}{4}$.

$$\begin{array}{r} 196 \\ 15\frac{3}{4} \\ \hline \frac{3}{4} \times 196 = 147 \\ 5 \times 196 = 980 \\ 10 \times 196 = 196 \\ \hline 15\frac{3}{4} \times 196 = 3087 \end{array}$$

46. Canceling is useful in finding products of fractions.

EXAMPLE: $\frac{2}{3} \times \frac{9}{16} \times \frac{8}{15} \times \frac{5}{23}$. By canceling, this becomes

$$\frac{\overset{3}{\cancel{2}}}{\underset{\cancel{2}}{3}} \times \frac{\overset{3}{\cancel{9}}}{\underset{\cancel{2}}{16}} \times \frac{\overset{2}{\cancel{8}}}{\underset{\cancel{3}}{15}} \times \frac{5}{23} = \frac{1}{1} \times \frac{3}{1} \times \frac{1}{1} \times \frac{2}{23} = \frac{6}{23}.$$

WRITTEN EXERCISES

Multiply (canceling when possible) :

- | | | | |
|--|--|---|---|
| 1. $4 \times \frac{3}{4}$. | 5. $\frac{3}{4} \times 48$. | 9. $\frac{3}{8} \times 18$. | 13. $16 \times 2\frac{1}{2}$. |
| 2. $\frac{3}{8} \times 5$. | 6. $\frac{5}{8} \times \frac{9}{16}$. | 10. $\frac{3}{8} \times \frac{5}{8}$. | 14. $7\frac{1}{2} \times 50\frac{1}{2}$. |
| 3. $\frac{4}{5} \times \frac{3}{5}$. | 7. $\frac{3}{16} \times \frac{4}{5}$. | 11. $\frac{5}{8} \times \frac{3}{10}$. | 15. $5\frac{1}{2} \times 12\frac{3}{4}$. |
| 4. $\frac{3}{5} \times \frac{7}{8}$. | 8. $\frac{9}{16} \times 4\frac{3}{4}$. | 12. $\frac{7}{8} \times \frac{9}{16}$. | 16. $80\frac{1}{2} \times 2\frac{5}{7}$. |
| 17. $\frac{4}{5} \times \frac{5}{8} \times \frac{9}{16} \times \frac{3}{10}$. | 24. $\frac{5}{8} \times \frac{7}{8} \times \frac{3}{8} \times \frac{4}{9}$. | | |
| 18. $1\frac{1}{2} \times 5\frac{1}{2} \times 7\frac{1}{2}$. | 25. $\frac{4}{5} \times 3\frac{3}{4} \times 5\frac{1}{2}$. | | |
| 19. $4\frac{1}{2} \times 7\frac{3}{10} \times 11\frac{1}{2}$. | 26. $40\frac{1}{2} \times 66\frac{2}{3} \times 15\frac{1}{2}$. | | |
| 20. $1\frac{1}{2} \times 2\frac{1}{2} \times 3\frac{1}{2}$. | 27. $\frac{7}{10} \times 30 \times 4\frac{1}{2}$. | | |
| 21. $\frac{3}{8} \times 1\frac{5}{8} \times 2\frac{7}{10}$. | 28. $16\frac{2}{3} \times 3\frac{3}{8} \times 1\frac{2}{3}$. | | |
| 22. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}$. | 29. $\frac{1}{2} \times \frac{4}{5} \times 2\frac{3}{10}$. | | |
| 23. $\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} \times \frac{4}{5}$. | 30. $27\frac{1}{2} \times 12\frac{2}{3} \times 6\frac{4}{11}$. | | |

Solve :

31. Find the product of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, and $\frac{4}{5}$.
32. From the product of $11\frac{1}{2}$ and $2\frac{3}{4}$ take the product of $6\frac{1}{2}$ and $3\frac{3}{10}$.
33. To the product of $87\frac{3}{4}$ and $5\frac{7}{8}$ add the product of $46\frac{1}{2}$ and $4\frac{1}{2}$.
34. To $16\frac{1}{2}$ add $11\frac{3}{4}$, and multiply the result by $12\frac{1}{2}$.
35. From $18\frac{1}{2}$ subtract $6\frac{1}{2}$, and multiply the result by $7\frac{1}{2}$.
36. To $11\frac{3}{4}$ add the product of $6\frac{1}{2}$ and $7\frac{3}{4}$, and multiply the result by $3\frac{3}{8}$.
37. Which is greater and how much, $82\frac{1}{2} \times 63\frac{1}{4}$ or $123\frac{1}{2} \times 39\frac{1}{2}$?
38. Rose has $\frac{3}{4}$ of a yard of ribbon and uses $\frac{1}{4}$ of this. What part of a yard does she use?
39. Find the cost of $5\frac{1}{2}$ yd. of cloth at $\frac{3}{4}$ of a dollar a yard.
40. A teamster drew 20 loads of $\frac{3}{4}$ of a cubic yard each every day for $4\frac{1}{2}$ days. How many cubic yards did he draw in all?

41. Some blocks of granite weighed $4\frac{2}{16}$ tons each. What was the weight of 128 blocks?

42. When cotton is worth $\$53\frac{3}{4}$ per bale, what is the value of 186 bales?

43. A garden was $16\frac{3}{4}$ yd. long and $5\frac{1}{4}$ yd. wide. How many square yards in the area of the garden?

44. How many miles does a train travel in $6\frac{3}{4}$ hr. at the rate of $57\frac{1}{4}$ mi. per hr.?

45. A rug $10\frac{1}{2}$ ft. long by $7\frac{3}{4}$ ft. wide is laid on the floor of a room $14\frac{1}{2}$ ft. long by $12\frac{1}{2}$ ft. wide. How many square feet in the area of the part of the floor not covered by the rug?

46. How many tons of coal in a train of 56 coal cars bearing an average load of $46\frac{1}{2}$ tons each?

47. A man worked $18\frac{3}{4}$ hr. at $47\frac{1}{2}$ cts. per hour. How much did he earn?

48. A schoolroom has 6 rows of desks each $2\frac{5}{8}$ ft. wide; the five aisles between the desks are each $3\frac{1}{4}$ ft. wide, and the two outer aisles are each $4\frac{1}{4}$ ft. wide. Find the width of the room.

49. Each row of desks of Exercise 48 contained ten desks. The distance from the front of each desk to the front of the next one back of it was $4\frac{1}{2}$ ft.; the distance from the front of the tenth desk to the rear wall was $7\frac{3}{4}$ ft., and that from the front wall to the front of the first desk was $12\frac{5}{8}$ ft. Find the length of the schoolroom.

50. When a certain engineer shuts off steam and puts on the brakes, the driving wheels of his engine still revolve $16\frac{1}{2}$ times before the engine comes to a standstill. If the driving wheels are $20\frac{3}{4}$ ft. in circumference, how far from the place where the brakes were applied is the engine stopped?

51. A drawing was made on the scale of $\frac{3}{32}$; that is, every length on the plan was $\frac{3}{32}$ of the actual length in the building. Find the lengths in the drawing corresponding to the following lengths in the building: $40\frac{1}{2}$ ft.; $30\frac{1}{2}$ ft.; $25\frac{1}{2}$ ft.; $20\frac{3}{4}$ ft.

52. Mr. Morgan received $\frac{3}{4}$ of the income of a farm; he spent $\frac{5}{12}$ of his share. What part of the whole income from the farm did he spend?

53. The coffee berry grows on a tree. The average yield of a coffee tree is 6 lb. of salable coffee in one season; the average productive life of a tree is 30 yr. Find the production of 1 coffee tree during its life, and the value of the coffee at $15\frac{1}{2}$ cts. a pound.

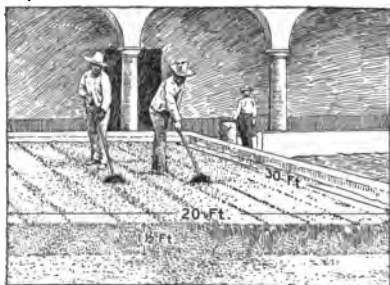
54. What would be the value of the coffee from 48,000 such trees in one season at $14\frac{1}{2}$ cts. a pound?

55. Different countries produce different grades of coffee:

COUNTRY.	WHOLESALE PRICE.	COUNTRY.	WHOLESALE PRICE.
Mexico.....	$15\frac{1}{2}\phi$	Venezuela.....	$13\frac{1}{2}\phi$
Costa Rica.....	$14\frac{1}{2}$	Colombia.....	$13\frac{1}{2}$
Guatemala.....	$14\frac{1}{2}$	Brazil.....	$9\frac{1}{2}$

What would the coffee mentioned in Exercise 54 be worth at the price named for each country?

56. The coffee crop of Brazil often amounts to 11,000,000 sacks of $132\frac{1}{2}$ lb. each. How many pounds is the yield?



57. What is the value of this product according to Exercise 55?

58. $70\frac{1}{2}$ lb. of coffee occupy about 1 cu. ft. How many pounds are there in the bed of dried coffee shown in the picture?

59. Venezuela averages 55,000 tons of coffee a year, $\frac{1}{10}$ of which is exported. What is the value of the exports according to Exercise 55?

DIVISION

47. *The quotient of any two fractions may be found by inverting the divisor and multiplying; or*

48. *The quotient of any two fractions may be found by changing them to fractions having a common denominator and then dividing the numerators of these fractions.*

Which of these processes is used in (1) below? In (2)?

$$(1) \frac{2}{3} \div \frac{4}{5} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12} = \frac{5}{6}.$$

$$(2) \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{5}{6}.$$

49. *To divide an integer by a fraction, write the integer with 1 as a denominator and proceed as in the case of two fractions.*

EXAMPLE: Divide 28 by $\frac{4}{7}$.

$$28 \div \frac{4}{7} = \frac{28}{1} \times \frac{7}{4} = 49, \text{ or } 49.$$

50. *To divide a fraction by an integer, divide the numerator or multiply the denominator by the integer.*

Thus, $\frac{3}{8} \div 4 = \frac{3}{32}$. Also, $\frac{3}{8} \div 5 = \frac{3}{40}$.

51. *To divide mixed numbers, change them to improper fractions and divide as in the case of two fractions.*

ORAL EXERCISES

Divide:

- | | | | |
|---------------------------|----------------------------|------------------------------|-----------------------------|
| 1. $\frac{1}{2} \div 5.$ | 6. $\frac{4}{5} \div 5.$ | 11. $\frac{3}{8} \div 7.$ | 16. $1\frac{2}{3} \div 6.$ |
| 2. $\frac{5}{6} \div 9.$ | 7. $\frac{7}{8} \div 8.$ | 12. $\frac{9}{10} \div 8.$ | 17. $2\frac{4}{7} \div 12.$ |
| 3. $\frac{8}{9} \div 4.$ | 8. $\frac{6}{7} \div 5.$ | 13. $\frac{60}{17} \div 5.$ | 18. $\frac{5}{6} \div 7.$ |
| 4. $\frac{5}{8} \div 4.$ | 9. $2\frac{1}{4} \div 3.$ | 14. $10\frac{2}{3} \div 10.$ | 19. $\frac{7}{8} \div 6.$ |
| 5. $\frac{7}{10} \div 5.$ | 10. $2\frac{2}{3} \div 3.$ | 15. $3\frac{1}{2} \div 6.$ | 20. $\frac{7}{8} \div 7.$ |

Solve:

21. How many breadths of carpet $\frac{3}{4}$ of a yard wide are needed to cover a room 15 ft. wide?

22. How many breadths of plush $1\frac{1}{2}$ yd. wide are needed to make a stage curtain 45 ft. wide?

23. How many yards are there in a roll of carpet costing \$6 at $\frac{2}{3}$ of a dollar a yard?

24. Into how many pieces $\frac{1}{2}$ of a yard long can 15 yd. of ribbon be divided?

25. How many strips of carpeting $\frac{3}{4}$ yd. wide and 8 yd. long are required to cover a room 6 yd. wide and 8 yd. long?

26. If a distance of 28 ft. is divided into equal intervals of $3\frac{1}{2}$ ft. each and a shrub planted in each interval, how many shrubs will be required?

27. If $2\frac{1}{2}$ ft. of ribbon are needed to make one rosette, how many rosettes can be made with 27 ft. of ribbon? With 30 yd. of ribbon?

WRITTEN EXERCISES

Divide:

- | | | | |
|--|--|---|--|
| 1. $\frac{1}{2} \div \frac{1}{3}$. | 14. $\frac{1}{2} \div \frac{2}{3}$. | 27. $\frac{2}{3} \div \frac{1}{3}$. | 40. $400 \div \frac{3}{20}$. |
| 2. $8 \div \frac{7}{16}$. | 15. $\frac{2}{3} \div \frac{1}{3}$. | 28. $\frac{1}{40} \div \frac{13}{80}$. | 41. $6\frac{2}{3} \div 4\frac{5}{6}$. |
| 3. $15 \div \frac{1}{16}$. | 16. $25 \div \frac{5}{8}$. | 29. $\frac{7}{84} \div \frac{1}{8}$. | 42. $\frac{70}{125} \div \frac{3}{5}$. |
| 4. $10 \div \frac{5}{64}$. | 17. $\frac{9}{16} \div \frac{5}{32}$. | 30. $\frac{50}{3} \div \frac{25}{6}$. | 43. $12\frac{1}{2} \div 3\frac{3}{20}$. |
| 5. $\frac{7}{8} \div \frac{1}{12}$. | 18. $16 \div \frac{1}{4}$. | 31. $\frac{2}{3} \div \frac{2}{3}$. | 44. $7\frac{1}{8} \div 12\frac{1}{4}$. |
| 6. $\frac{1}{16} \div \frac{5}{8}$. | 19. $25 \div \frac{1}{8}$. | 32. $50 \div \frac{3}{8}$. | 45. $900 \div \frac{9}{10}$. |
| 7. $\frac{5}{12} \div \frac{5}{8}$. | 20. $\frac{5}{12} \div \frac{3}{8}$. | 33. $18 \div \frac{3}{8}$. | 46. $9\frac{1}{10} \div 3\frac{3}{20}$. |
| 8. $\frac{9}{10} \div \frac{3}{100}$. | 21. $\frac{4}{7} \div \frac{1}{7}$. | 34. $40 \div \frac{7}{12}$. | 47. $6\frac{1}{25} \div 12\frac{3}{5}$. |
| 9. $\frac{2}{3} \div \frac{1}{4}$. | 22. $6 \div \frac{2}{3}$. | 35. $\frac{3}{8} \div \frac{1}{8}$. | 48. $20\frac{1}{2} \div 5\frac{3}{4}$. |
| 10. $\frac{1}{8} \div \frac{1}{8}$. | 23. $7 \div \frac{1}{8}$. | 36. $4\frac{2}{3} \div \frac{7}{8}$. | 49. $2\frac{1}{2} \div 2\frac{1}{3}$. |
| 11. $\frac{6}{7} \div \frac{9}{14}$. | 24. $9 \div \frac{1}{8}$. | 37. $\frac{3}{8} \div \frac{1}{8}$. | 50. $6\frac{1}{2} \div 2\frac{1}{3}$. |
| 12. $\frac{1}{8} \div \frac{8}{11}$. | 25. $\frac{1}{9} \div 9$. | 38. $\frac{3}{8} \div 3$. | 51. $5\frac{1}{2} \div 4\frac{2}{3}$. |
| 13. $\frac{7}{8} \div \frac{1}{8}$. | 26. $1 \div \frac{2}{3}$. | 39. $\frac{3}{8} \div \frac{3}{8}$. | 52. $2\frac{2}{3} \div 6\frac{1}{2}$. |

53. Divide the product of $6\frac{1}{2}$ and $1\frac{1}{3}$ by $\frac{4}{5}$.
54. Divide the product of $2\frac{1}{2}$ and $7\frac{1}{2}$ by that of $3\frac{1}{4}$ and $\frac{2}{3}$.
55. Divide the sum of $12\frac{1}{2}$ and $6\frac{1}{4}$ by the difference of $4\frac{1}{2}$ and $2\frac{1}{2}$.
56. Divide the sum of $18\frac{1}{2}$ and $9\frac{3}{4}$ by their difference.
57. Divide the difference of $9\frac{1}{2}$ and $2\frac{1}{2}$ by their product.
58. Divide the product of $8\frac{1}{2}$ and $6\frac{3}{4}$ by their sum.
59. Divide $4\frac{1}{2}$ more than the product of $6\frac{1}{2}$ and $5\frac{1}{2}$ by $3\frac{1}{2}$ more than the quotient of $7\frac{1}{2}$ divided by $2\frac{1}{2}$.

Solve :

60. How many pieces of cloth $4\frac{1}{2}$ yd. long can be cut from a bolt of cloth containing 806 yd.?

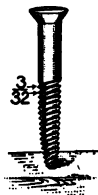
61. How many pieces of ribbon $3\frac{1}{16}$ ft. long can be cut from a bolt of ribbon 100 yd. long? What will be the length of the piece that is left?

62. A train runs $628\frac{3}{4}$ mi. in $28\frac{1}{2}$ hr. What is its average rate of speed per hour?

63. How many books $1\frac{1}{2}$ in. thick can be set on a shelf 30 in. long?

64. How many wagonloads of $2\frac{3}{8}$ tons each will be required to haul away 600 tons of sand? What will be the weight of the last load?

65. A certain screw is $3\frac{1}{4}$ in. long, the threads are $\frac{3}{32}$ in. apart; in one revolution the screw is driven a distance equal to that between two threads. How many revolutions must be made to drive the screw its whole length?



66. A screw is $2\frac{1}{2}$ in. long, and the threads are $\frac{1}{8}$ in. apart. How many complete turns are required to drive it in?

67. A screw has threads $\frac{5}{16}$ in. apart. How many turns are needed to drive it $2\frac{1}{2}$ in.?

COMPLEX FRACTIONS

52. If a fraction occurs in the numerator or denominator of a fraction or in both, the latter fraction is called a **complex fraction**.

Thus, $\frac{1\frac{1}{2}}{5}$, $\frac{3}{5\frac{1}{2}}$, $\frac{3\frac{1}{2}}{5\frac{3}{4}}$, $\frac{6 - \frac{1}{2}}{12\frac{1}{2} + 4\frac{3}{4}}$ are complex fractions.

Such fractions furnish drill in operations, but they have no place in practical arithmetic.

53. *Complex fractions are simplified by performing the operations indicated in the numerator and the denominator, and then reducing the fraction to lower terms if possible.*

EXAMPLES:

1. Simplify $\frac{5\frac{3}{4}}{6\frac{1}{2}}$.

$$(1) 5\frac{3}{4} = \frac{11}{2}.$$

$$(3) \frac{11}{2} \div \frac{13}{2} = \frac{11}{13}.$$

$$(2) 6\frac{1}{2} = \frac{13}{2}.$$

$$(4) \text{Therefore, } \frac{5\frac{3}{4}}{6\frac{1}{2}} = \frac{11}{13}.$$

2. Simplify $\frac{6 - \frac{1}{2}}{12\frac{1}{2} + 4\frac{3}{4}}$.

$$(1) 6 - \frac{1}{2} = 5\frac{1}{2} = \frac{11}{2}.$$

$$(3) \frac{11}{2} \div \frac{254}{16} = \frac{118}{127}.$$

$$(2) 12\frac{1}{2} + 4\frac{3}{4} = 16\frac{1}{2} = \frac{33}{2}.$$

$$(4) \text{Therefore, } \frac{6 - \frac{1}{2}}{12\frac{1}{2} + 4\frac{3}{4}} = \frac{118}{127}.$$

WRITTEN EXERCISES

Simplify:

1. $\frac{2\frac{3}{4}}{8\frac{5}{8}}$.

4. $\frac{5\frac{3}{4}}{8\frac{9}{10}}$.

7. $\frac{126\frac{11}{10}}{42\frac{3}{5}}$.

2. $\frac{4 \times 3\frac{1}{2}}{7 \div 2\frac{1}{2}}$.

5. $\frac{7 - 6\frac{3}{4}}{\frac{2}{3} \text{ of } \frac{3}{4}}$.

8. $\frac{7.6 + 3\frac{1}{2}}{9 - 6\frac{3}{4}}$.

3. $\frac{5\frac{1}{2} \div 7\frac{1}{2}}{6\frac{1}{2} \times 3\frac{1}{2}}$.

6. $\frac{40\frac{9}{10}}{20\frac{3}{5}}$.

9. $\frac{9 - 3\frac{3}{4} + 7\frac{1}{2}}{\frac{1}{3} \text{ of } \frac{7}{12}}$.

REVIEW

ORAL EXERCISES

1. What principle is employed when a fraction is changed to an equal fraction having a different denominator? Illustrate.

2. What is a fractional unit? Illustrate your answer.

3. How must fractions be expressed before they can be added or subtracted?

4. What is meant by the least common denominator of several fractions? Illustrate.

5. What is the least common denominator of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{4}$? Of $\frac{1}{12}$, $\frac{3}{8}$, $\frac{5}{6}$?

6. What is a mixed number? Explain two ways of adding mixed numbers. Illustrate each way.

7. Explain two ways of subtracting mixed numbers.

8. How may one fraction be multiplied by another?

9. What is the product of $\frac{2}{3}$ and $\frac{4}{5}$? Of $\frac{3}{4}$ and $\frac{5}{6}$? Of $\frac{4}{5}$ and $\frac{3}{4}$?

10. How may one fraction be divided by another? State another way.

Add:

11. $\frac{5}{8} + \frac{2}{5}$.

15. $\frac{4}{5} + \frac{3}{10}$.

19. $\frac{1}{2} + 1\frac{1}{2} + \frac{3}{4}$.

12. $\frac{1}{2} + \frac{1}{3} + \frac{5}{6}$.

16. $\frac{7}{8} + \frac{3}{4}$.

20. $\frac{1}{4} + 1\frac{1}{4} + 2\frac{5}{8}$.

13. $\frac{6}{8} + \frac{4}{5}$.

17. $\frac{5}{6} + \frac{7}{12}$.

21. $\frac{1}{2} + \frac{1}{3} + \frac{5}{6}$.

14. $\frac{3}{4} + \frac{1}{2}$.

18. $\frac{1}{2} + \frac{7}{10}$.

22. $\frac{1}{3} + \frac{1}{4} + 1\frac{1}{2}$.

Subtract:

23. $\frac{5}{8} - \frac{3}{5}$.

27. $\frac{7}{8} - \frac{1}{5}$.

31. $\frac{4}{5} - \frac{1}{3}$.

24. $\frac{9}{10} - \frac{2}{3}$.

28. $\frac{2}{3} - \frac{2}{5}$.

32. $\frac{7}{12} - \frac{1}{3}$.

25. $\frac{7}{8} - \frac{1}{4}$.

29. $\frac{4}{5} - \frac{1}{3}$.

33. $\frac{7}{10} - \frac{3}{5}$.

26. $1\frac{1}{2} - \frac{2}{3}$.

30. $\frac{1}{4} - \frac{1}{6}$.

34. $1\frac{1}{3} - \frac{1}{2}$.

Multiply:

35. $\frac{3}{8} \times \frac{2}{3}$.

36. $\frac{3}{4} \times \frac{5}{8}$.

37. $3\frac{3}{4} \times \frac{1}{2}$.

38. $\frac{7}{8} \times \frac{9}{14}$.

39. $\frac{5}{12} \times \frac{6}{15}$.

40. $1\frac{1}{2} \times \frac{3}{4}$.

41. $\frac{7}{8} \times \frac{2}{3}$.

42. $2\frac{1}{2} \times \frac{2}{3}$.

43. $\frac{3}{2} \times \frac{4}{5}$.

44. $1\frac{3}{4} \times \frac{2}{3}$.

45. $\frac{7}{12} \times \frac{5}{8}$.

46. $\frac{4}{5} \times \frac{8}{9}$.

47. $6 \times 2\frac{1}{3}$.

48. $\frac{4}{5} \times \frac{4}{5}$.

49. $\frac{4}{5} \times \frac{5}{4}$.

Divide:

50. $\frac{8}{9} \div \frac{2}{3}$.

51. $\frac{5}{6} \div \frac{3}{4}$.

52. $4 \div \frac{2}{3}$.

53. $\frac{3}{8} \div 6$.

54. $\frac{7}{8} \div \frac{3}{4}$.

55. $\frac{5}{12} \div \frac{5}{8}$.

56. $8 \div 1\frac{1}{5}$.

57. $\frac{3}{4} \div \frac{4}{5}$.

58. $1\frac{1}{2} \div \frac{2}{3}$.

59. $\frac{5}{8} \div \frac{1}{4}$.

60. $\frac{4}{5} \div 1\frac{2}{3}$.

61. $\frac{7}{4} \div 1\frac{2}{3}$.

62. On a certain day the following were the prices of grain per bushel on the New York market. The months indicate when the grain is to be delivered to the purchaser. The prices are those prevailing at the opening and at the closing of the market; also the highest and the lowest on that day.

WHEAT	Opening.	Highest.	Lowest.	Closing.
May.....	95	95	95	95
July.....	91 $\frac{1}{2}$	91 $\frac{1}{4}$	91 $\frac{1}{8}$	91 $\frac{5}{8}$
Sept.....	83 $\frac{7}{8}$	84 $\frac{1}{8}$	83 $\frac{1}{4}$	83 $\frac{7}{8}$
Dec.....	84	84	83 $\frac{1}{2}$	83 $\frac{3}{4}$
CORN				
May.....	59 $\frac{3}{4}$	60 $\frac{1}{2}$	59 $\frac{1}{4}$	60
July.....	53 $\frac{3}{8}$	53 $\frac{7}{8}$	53 $\frac{1}{4}$	53 $\frac{7}{8}$
Sept.....	52 $\frac{1}{4}$	52 $\frac{3}{8}$	52 $\frac{1}{8}$	52 $\frac{3}{4}$

What was the difference between the opening and closing prices of July wheat? Between the highest and lowest prices?

63. Answer the same questions for September wheat.

64. How much was gained on 1000 bu. of July wheat bought at the opening price and sold at the closing price?

65. Make and solve five other problems from the prices in the table above.

WRITTEN EXERCISES

Perform the operations and simplify:

1. $1\frac{1}{2} + \frac{5}{8} + \frac{3}{8} + \frac{7}{12}$.
2. $4\frac{1}{4} + \frac{7}{8} + \frac{3}{4} + \frac{9}{16}$.
3. $2\frac{3}{4} + \frac{5}{12} + \frac{1}{6} - 1\frac{1}{2} - \frac{5}{8}$.
4. $\frac{3}{8} + 5\frac{9}{10} - 3\frac{1}{2} - 1\frac{3}{8}$.
5. $9 - 1\frac{1}{2} - 2\frac{3}{4} - 3\frac{7}{8}$.
6. $8\frac{3}{8} + 3\frac{1}{4} + 6\frac{7}{12} + 5\frac{1}{2}$.
7. $11\frac{1}{2} + 3\frac{5}{8} - 4\frac{3}{8} + 7\frac{1}{8}$.
8. $3\frac{3}{4} + 1\frac{1}{2} - \frac{5}{12} - \frac{3}{8}$.
9. $5\frac{1}{2} + 6\frac{3}{8} - \frac{5}{8} - 2\frac{1}{8}$.
10. $4\frac{9}{10} + 3\frac{2}{15} - 1\frac{7}{10} - 1\frac{1}{5}$.
11. $11\frac{2}{3} - 2\frac{4}{3} + 3\frac{2}{3} - 3\frac{7}{3}$.
12. $\frac{1}{2}\frac{3}{4} + \frac{1}{2}\frac{5}{4} - \frac{7}{4} + 1\frac{9}{4}$.
13. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$.
14. $\frac{1}{2} + \frac{2}{3} - \frac{4}{6} + \frac{7}{10} - 1\frac{1}{5}$.

Multiply:

15. $\frac{7}{8} \times \frac{3}{4}$.
16. $\frac{5}{8} \times \frac{6}{16}$.
17. $\frac{5}{8} \times \frac{3}{8}$.
18. $5\frac{1}{2} \times 8\frac{1}{4}$.
19. $7\frac{1}{2} \times 14\frac{1}{4}$.
20. $\frac{1}{4} \times \frac{1}{4}$.
21. $\frac{7}{12} \times \frac{3}{16}$.
22. $\frac{1}{8} \times \frac{1}{8}$.
23. $8\frac{1}{2} \times 2\frac{3}{8}$.
24. $19\frac{7}{16} \times 21\frac{3}{8}$.
25. $43\frac{7}{16} \times 20\frac{3}{8}$.
26. $1\frac{2}{3} \times 1\frac{1}{3}$.
27. $\frac{1}{2} \times \frac{7}{12}$.
28. $\frac{1}{8} \times \frac{1}{16}$.
29. $12\frac{1}{4} \times 8\frac{7}{12}$.
30. $20\frac{1}{16} \times 15\frac{1}{8}$.
31. $14\frac{3}{8} \times 5\frac{2}{11}$.
32. $27\frac{3}{8} \times 8\frac{1}{8}$.

Divide:

33. $\frac{3}{8} \div \frac{1}{6}$.
34. $\frac{7}{8} \div \frac{3}{8}$.
35. $\frac{2}{3} \div \frac{1}{3}$.
36. $5\frac{1}{8} \div 3\frac{1}{4}$.
37. $\frac{1}{8} \div \frac{5}{8}$.
38. $8\frac{1}{2} \div 1\frac{1}{2}$.
39. $\frac{5}{8} \div \frac{1}{10}$.
40. $6\frac{3}{8} \div 7\frac{1}{2}$.
41. $\frac{1}{2} \div \frac{2}{4}$.
42. $\frac{1}{6} \div \frac{2}{3}$.
43. $4\frac{1}{2} \div 3\frac{1}{2}$.
44. $19\frac{6}{15} \div 4\frac{3}{5}$.
45. $\frac{3}{8} \div \frac{1}{10}$.
46. $\frac{5}{8} \div \frac{1}{4}$.
47. $5\frac{3}{8} \div 8\frac{1}{4}$.
48. $\frac{1}{6} \div \frac{3}{16}$.
49. $7\frac{1}{2} \div 2\frac{3}{8}$.
50. $40\frac{7}{12} \div 10\frac{3}{4}$.
51. Find the product of $2\frac{1}{2} + 5\frac{5}{8} - \frac{3}{4}$ and $2\frac{1}{2} \div 4\frac{1}{2}$.
52. If the dividend is $\frac{7}{16} + 1\frac{3}{4} - \frac{5}{8}$ and the divisor is $\frac{5}{8} \times 2\frac{3}{4}$, what is the quotient in simplest form?
53. Find the product of $4\frac{1}{2} + 2\frac{3}{8} - 3\frac{1}{8}$ and $3\frac{3}{8} \div 2\frac{1}{2}$.
54. Find the simplest form of the quotient if the dividend is $5\frac{1}{2} - 1\frac{7}{8} + 3\frac{1}{4}$ and the divisor is $4\frac{1}{8} \times 7\frac{3}{4}$.
55. Add $7\frac{1}{2} \times 6\frac{2}{11}$ to $41\frac{1}{4} \div 4\frac{1}{11}$.

Solve:

56. A rug $8\frac{1}{4}$ ft. by $10\frac{1}{4}$ ft. is placed on a floor $10\frac{3}{4}$ ft. by 13 ft. What is the area of the uncovered part of the floor?

57. What is the cost of a metal ceiling $18\frac{1}{2}$ yd. long and $10\frac{1}{4}$ yd. wide at \$1.87 per sq. yd.?

58. A factory burned $18\frac{1}{10}$ tons of coal in January, $17\frac{1}{4}$ tons in February, $19\frac{3}{8}$ tons in March, $17\frac{7}{8}$ tons in April. How many did it burn in all? What did the coal cost at $\$6\frac{1}{4}$ per ton?

59. What is the cost of covering a wall $14\frac{1}{2}$ yd. long and $3\frac{1}{2}$ yd. high with burlap, if the burlap costs $27\frac{1}{2}$ cts. per square yard, and the hanging $12\frac{1}{2}$ cts. per square yard?

60. What is the cost of putting up a picture molding around a room $24\frac{3}{4}$ ft. long and $16\frac{1}{4}$ ft. wide, if the molding costs $5\frac{1}{2}$ cts. per running foot, and the carpenter spends $4\frac{3}{4}$ hr. in putting it up at $42\frac{1}{2}$ cts. per hour?

61. A $17\frac{1}{4}$ -acre field of corn yielded 48 bu. to the acre. At $62\frac{1}{2}$ cts. per bushel, what was the value of the yield? Various expenses amounted to $\frac{1}{10}$ of this sum. What was the profit?

62. If wheat had been grown in the field of Exercise 61, the profit would have been $\frac{1}{2}$ more. Find the profit from wheat.

63. What is the cost of a piece of wire netting $2\frac{1}{2}$ ft. wide and $63\frac{3}{4}$ ft. long at $\frac{3}{8}$ ct. per square foot?

64. What is the cost of sodding a lawn $27\frac{1}{2}$ yd. long and $19\frac{1}{2}$ yd. wide at $7\frac{1}{2}$ cts. per square yard?

65. What is the cost of planting a line of bulbs around the lawn of Exercise 64, if an average of 27 bulbs at $\frac{3}{8}$ ct. each are planted per yard?

66. A half-eagle, or 5-dollar gold piece, weighs 129 grains; $\frac{9}{10}$ of its weight is pure gold. How many grains of gold are there in a half-eagle? How many grains of cheaper metal?

67. The 5-cent piece weighs 77.16 grains; $\frac{1}{4}$ of each coin is nickel, and the rest copper. How many grains of each kind are there in a 5-cent piece?

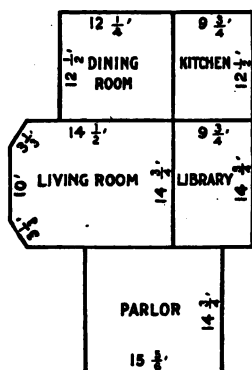
68. Find the number of feet of baseboard needed for each room in the plan, making no allowance for openings.

69. How many feet more are needed for the parlor than for the library?

70. Find the total length of the house, counting the thickness of each wall (outer or partition) as $\frac{1}{2}$ ft.

71. Make and solve 3 other problems about the plan.

72. The silver dollar weighs 412.5 grains. $\frac{2}{10}$ of its weight is pure silver. How many grains of pure silver are there in one dollar?



73. A builder bought a lot in New York City, and later found that the lot did not include a narrow strip along the street. This strip was $\frac{7}{8}$ in. wide and 40 ft. long. What was its area in square inches? He bought the strip for \$50. How much was this per square inch? Per square foot?

74. At the rate found in Exercise 73, what would be the cost of a city lot 25 ft. by 100 ft.?

75. To allow for shrinkage, patterns for cast-iron articles are made $\frac{1}{8}$ in. longer *per foot* than the article. Similarly, for width and thickness. What are the length and the width of a pattern for a cast-iron article $3\frac{1}{2}$ ft. long and 8 in. wide?

76. The allowance per foot for shrinkage in other castings is:

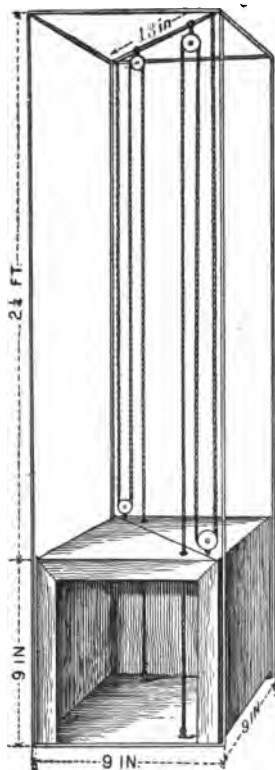
Brass	$\frac{3}{16}$ in.	Tin	$\frac{1}{16}$ in.
Lead	$\frac{1}{8}$ in.	Zinc	$\frac{3}{16}$ in.

A casting of lead is to be 18 in. long and 15 in. wide. What are the length and the breadth of the pattern?

77. Make and solve 4 other problems about patterns for castings.

78. Each of 15 pupils in manual training made a weaving loom. The base frame of each loom required two pieces of molding $1\frac{1}{2}$ ft. long and two pieces $\frac{3}{4}$ ft. long. How many feet were used for all of the looms? How many feet were left out of six 12-foot strips?

79. Harold made a model of an elevator; the framework was made of stiff wire. Find from the given dimensions how many feet of wire were used. Find how many feet of string he used for the cables.



80. He noticed that he worked on the model $1\frac{1}{2}$ hr. on each of 3 days, $1\frac{1}{2}$ hr. on each of 2 days, and $\frac{3}{4}$ hr. on each of 5 days. How long did it take him to make it? How many days of 8 hr. each?

81. A class in domestic science made 16 cup-custards. For each cup they allowed 1 egg, $1\frac{1}{2}$ oz. of sugar, and $\frac{1}{2}$ pt. of milk. At 18 cts. a dozen for eggs, $5\frac{1}{2}$ cts. a pound for sugar, and 6 cts. a quart for milk, what did all of these materials cost?

82. A class of girls made 5 sachet bags, 8 calendars, and 7 letter cases for an exhibition. Each sachet bag required $1\frac{1}{2}$ ft. of ribbon, each calendar $\frac{3}{4}$ ft., and each letter case $1\frac{3}{4}$ ft. How many feet of ribbon did they use? How many yards?

83. The boys made bent iron candlesticks, each requiring $2\frac{3}{4}$ ft. of iron. How many feet of iron did they use in making 6 candlesticks?

CHAPTER III

DENOMINATE NUMBERS

54. Numbers considered apart from particular things are called **abstract numbers**. Numbers that refer to particular things are called **concrete numbers**.

Thus, 2, 3, $\frac{4}{5}$, and $1\frac{1}{2}$ are abstract numbers; while 2 words, 3 apples, $\frac{4}{5}$ ft., and $\$1\frac{1}{2}$ are concrete numbers.

55. Concrete numbers used in weights and measures are commonly called **denominate numbers**.

Thus, 8 lb., 12 gal. 3 qt., 2 bu. 2 pk. 6 qt., and \$3.75 are denominate numbers.

The tables of denominate numbers are given on pages 235 and 236.

REDUCTION

56. In solving problems it is often necessary to reduce a denominate number of several denominations to one number of lower denomination. This is called **reduction descending**.

EXAMPLE: How many lamps holding 1 pt. can be filled from 4 gal. 3 qt. 1 pt. of kerosene?

Solution: (1) 4 gal. = 4×4 qt., or 16 qt.

(2) 16 qt. + 3 qt. = 19 qt.

(3) 19 qt. = 19×2 pt., or 38 pt.

(4) 38 pt. + 1 pt. = 39 pt.

(5) Therefore, 4 gal. 3 qt. 1 pt. = 39 pt.

The problem now becomes "How many lamps holding 1 pt. can be filled from 39 pt.?" The result is evidently 39 lamps.

57. *A number of several denominations may be reduced to one of the lowest denomination by beginning with the highest, multiplying this by the number of units in the next lower denomination, adding to this result any such units that the number may contain, and repeating the process to the lowest denomination.*

WRITTEN EXERCISES

Reduce to the lowest denominations :

- | | |
|-----------------------------------|-------------------------------------|
| 1. 5 T. 200 lb. 8 oz. | 7. 5 da. 20 hr. 6 min. 30 sec. |
| 2. 17 gal. 3 qt. 1 pt. | 8. $40^{\circ} 20' 45''$. |
| 3. 6 gross 9 doz. 10 units. | 9. 12 bu. 3 pk. 7 qt. |
| 4. 5 cu. ft. and 1,020 cu. in. | 10. 10 rd. 12 ft. 8 in. |
| 5. 50 cu. yd. 8 cu. ft. 5 cu. in. | 11. 4 reams, 20 quires, 18 sheets. |
| 6. 1 mi. 160 rd. 2 ft. | 12. 10 sq. yd. 8 sq. ft. 40 sq. in. |

58. In solving problems containing denominate numbers, it is often necessary to change from lower to higher denominations. This is called **reduction ascending**.

EXAMPLE : Clarence picked 205 qt. of cherries. How many bushels and quarts did he pick?

Here it is necessary to express 205 qt. as bushels and quarts.

Solution : (1) 1 bu. = 32 qt.

(2) $205 \div 32 = 6$ and 13 remaining.

(3) Therefore, 205 qt. = 6 bu. and 13 qt.

Another plan is this:

(1) 1 qt. = $\frac{1}{32}$ bu.

(2) $205 \text{ qt.} = 205 \times \frac{1}{32} \text{ bu.} = 6\frac{13}{32} \text{ bu.} = 6 \text{ bu. } 13 \text{ qt.}$

The second solution is preferable if the result is wanted in one denomination, as $6\frac{13}{32}$ bu.

59. *A number of one denomination may be reduced to the next higher denomination by dividing it by the number of the given units that make one of the next denomination.*

WRITTEN EXERCISES

1. A milk wagon carried 125 pint-bottles. How many gallons, quarts, and pints was this?
2. An oil tank contained 16,526 pt. How many gallons, quarts, and pints was this?
3. A load of berries contained 327 one-quart boxes. How many bushels, pecks, and quarts in the load?

Express :

4. 875 pt. as gallons, quarts, pints.
5. 986 pt. as bushels and quarts.
6. 4,956 lb. as tons and pounds.
7. 4,763 min. as days, hours, minutes.
8. 5,690 sec. as hours, minutes, seconds.
9. 15,760 gr. avoirdupois as pounds, ounces, grains.
10. 4,325 in. as rods, feet, inches.
11. 10,378 ft. as miles and feet.
12. 5,890 sq. in. as square yards, square feet, square inches.
13. 15,763 cu. in. as cubic yards, cubic feet, cubic inches.

Reduction with Decimals

60. The custom is growing of expressing quantities in one denomination only, using fractions if necessary. Decimal fractions are usually the most convenient.

Thus, 1.5 ft. would be used instead of 1 ft. 6 in.

EXAMPLES :

1. How many hours, minutes, and seconds are there in .48 da.?

Solution : (1) $.48 \times 24 \text{ hr.} = 11.52 \text{ hr.}$

(2) $.52 \times 60 \text{ min.} = 31.2 \text{ min.}$

(3) $.2 \times 60 \text{ sec.} = 12 \text{ sec.}$

(4) Therefore, .48 da. = 11 hr. 31 min. 12 sec.

2. 1 ft. is what part of a yard? Express decimally as parts of a yard: 1 ft.; 2 ft.; 4 ft.; $1\frac{1}{2}$ ft.; $\frac{3}{4}$ ft.; $\frac{1}{2}$ ft.

3. Express decimally 7 yd. 2 ft. 11 in. as a number of yards.

Solution: (1) $11 \times \frac{1}{36}$ ft. = .92 ft.

(2) 2 ft. + .92 ft. = 2.92 ft.

(3) $2.92 \times \frac{1}{3}$ yd. = .97 yd.

(4) 7 yd. + .97 yd. = 7.97 yd.

(5) Therefore, 7 yd. 2 ft. 11 in. = 7.97 yd.

WRITTEN EXERCISES

1. Express in hours and minutes: 1.3 hr.; 2.7 hr.; 4.9 hr.; 12.6 hr.; .5 hr.

2. Add the numbers as they are given in Exercise 1. Also when changed to hours and minutes. Which way is shorter?

3. How many yards, feet, and inches are there in 7.8 yd.? In 5.25 yd.?

4. How many square rods are there in 1.7 acre?

5. How many square feet are there in 9.25 sq. yd.?

6. How many yards are there in 3.75 in.?

7. How many ounces are there in $12.37\frac{1}{2}$ lb.?

8. Express decimally, as a number of feet: 5 ft. 8 in.; 9 ft. 5 in.

9. Express decimally as gallons: 2 gal. 3 qt. 1 pt.; 5 gal. 1 qt. 1 pt.

10. Express decimally as square feet: 6 sq. ft. 72 sq. in.; 15 sq. ft. 96 sq. in.

11. Express decimally as square yards: 1 sq. yd. 8 sq. ft. 16 sq. in.

12. Express decimally as cubic yards: 4 cu. yd. 9 cu. ft. 144 cu. in.

13. Express decimally: $6\frac{3}{4}$ ft.; $2\frac{1}{2}$ ft.; $7\frac{1}{2}$ ft.; $8\frac{3}{4}$ ft.

14. Add the numbers of Exercise 13 as given. Add the numbers expressed decimally. Which way is the shorter?

ADDITION AND SUBTRACTION

61. EXAMPLE: Add 120 lb. 12 oz., 119 lb. 4 oz., and 118 lb. 8 oz.

Add the numbers in the first column at the right.	120 lb. 12 oz.
How many pounds and how many ounces	119 4
besides is this? Add the second column, including the 1 lb. from the first.	118 8
	<hr/> 358 lb. 8 oz.

62. *To add denominate numbers, arrange the addends in columns of like denomination and add the numbers in each column. If the sum in any column is equal to, or greater than, the number of units in the next higher denomination, reduce it and add the higher units to the next column.*

EXAMPLE: Subtract 17 hr. 54 min. from 25 hr. 36 min.

Add 1 hr. to the 36 min. of the right column and	25 hr. 36 min.
subtract 54 min. Take 1 hr. from 25 hr. of the	17 54
left column and subtract 17 hr. from 24 hr.	<hr/> 7 hr. 42 min.

63. *To subtract denominate numbers, arrange the numbers in columns of like denomination, and subtract. Where the minuend in any denomination is smaller than the subtrahend, increase it by the equivalent of one unit of the next higher denomination.*

WRITTEN EXERCISES

Add:

1. 7 gal. 3 qt. 1 pt.

8	2	
7	1	1
<hr/>		

2. 6 pk. 4 qt. 1 pt.

3	2	
6	7	1
<hr/>		

3. 10 mi. 80 rd. 10 ft.

20	20	15
15	100	8
<hr/>		

4. 7 yd. 2 ft. 2 in.

10	1	10
15		8
<hr/>		

$$\begin{array}{r} 5. \text{ 12 da. 10 hr. 15 min. 5 sec.} \\ \underline{18 \quad 20 \quad 45 \quad 55} \end{array}$$

$$\begin{array}{r} 8. \text{ 40 A. 80 sq. rd. 3 sq. yd.} \\ \underline{60 \quad 100 \quad 10} \end{array}$$

$$\begin{array}{r} 6. \text{ 40 bu. 3 pk. 6 qt.} \\ \underline{8 \quad 2 \quad 5} \\ 20 \quad 1 \quad 3 \end{array}$$

$$\begin{array}{r} 9. \text{ 4 wk. 6 da. 20 hr.} \\ \underline{12 \quad 3 \quad 10} \\ 15 \quad 5 \quad 12 \end{array}$$

$$\begin{array}{r} 7. \text{ 3 cu. yd. 20 cu. ft. 10 cu. in.} \\ \underline{10 \quad 18 \quad 100} \\ 15 \quad 12 \quad 1000 \end{array}$$

$$\begin{array}{r} 10. \text{ 30 sq. yd. 83 sq. ft. 40 sq. in.} \\ \underline{15 \quad 6 \quad 25} \\ 10 \quad 5 \quad 100 \end{array}$$

Subtract :

$$\begin{array}{r} 11. \text{ 15 wk. 5 da. 12 hr.} \\ \underline{4 \quad 6 \quad 20} \end{array}$$

$$\begin{array}{r} 14. \text{ 15 cu. yd. 12 cu. ft. 100 cu. in.} \\ \underline{12 \quad 18 \quad 1000} \end{array}$$

$$\begin{array}{r} 12. \text{ 20 mi. 20 rd. 15 ft.} \\ \underline{10 \quad 80 \quad 10} \end{array}$$

$$\begin{array}{r} 15. \text{ 40 bu. 3 pk. 6 qt.} \\ \underline{15 \quad 1 \quad 7} \end{array}$$

$$\begin{array}{r} 13. \text{ 15 yd. 2 ft. 2 in.} \\ \underline{7 \quad 1 \quad 10} \end{array}$$

$$\begin{array}{r} 16. \text{ 30 sq. yd. 5 sq. ft. 25 sq. in.} \\ \underline{15 \quad 6 \quad 100} \end{array}$$

Solve :

17. A lot has a frontage of 18 yd. 2 ft. 6 in., and another a frontage of 19 yd. 1 ft. 9 in. Find the total frontage.

18. The Empire State Express traveled from Rome to Albany in 2 hr. 49 min. 36 sec. and from Albany to New York City in 2 hr. 58 min. 50 sec. How long was it in traveling from Rome to New York?

19. An eclipse began at 8 hr. 38 min. 6 sec. P.M. and ended at 8 hr. 55 min. 51 sec. P.M. How long did it last?

20. From a 10-gallon can of milk 21 qt. and 1 pt. were sold. How much was left?

21. The best watches permit one to observe fifths of a second. A bicycle rider began a mile at 2 hr. 17 min. $23\frac{4}{5}$ sec. P.M. and ended it at 2 hr. 19 min. $56\frac{4}{5}$ sec. P.M. How long did it take him to ride the mile?

MULTIPLICATION

64. EXAMPLE :

A square lot measures 5 yd. 2 ft. 7 in. on a side. What is its perimeter?

- | | |
|---------------------------|--|
| 5 yd. 2 ft. 7 in. | (1) 4×7 in. = 28 in. = 2 ft. 4 in. |
| 4 | (2) 4×2 ft. = 8 ft. |
| <hr/> 20 yd. 8 ft. 28 in. | (3) 8 ft. + 2 ft. = 10 ft. = 3 yd. 1 ft. |
| 23 yd. 1 ft. 4 in. | (4) 4×5 yd. = 20 yd. |
| | (5) 20 yd. + 3 yd. = 23 yd. |
| | (6) Therefore, the perimeter is 23 yd. 1 ft. 4 in. |

65. *To multiply a compound number, multiply each denomination separately and reduce the results to units of higher denomination, if possible.*

WRITTEN EXERCISES

Multiply :

- | | |
|---------------------------------------|--|
| 1. 4 gal. 3 qt. 1 pt.
<hr/> 5 | 5. 10 sq. rd. 20 sq. ft. 12 sq. in.
<hr/> 20 |
| 2. 3 pk. 7.5 qt. .5 pt.
<hr/> 15 | 6. 3 cu. yd. 20 cu. ft. 12 cu. in.
<hr/> 3.25 |
| 3. 2 bu. 6 pk. 1.5 qt.
<hr/> 10 | 7. 40 hr. 20 min. 16 sec.
<hr/> 2.5 |
| 4. 10 mi. 40.5 rd. 12 ft.
<hr/> 16 | 8. 40 A. 12 sq. rd. 5 sq. yd.
<hr/> 62 |

9. Multiply 5 yd. 2 ft. 7 in. by 4. Then express 5 yd. 2 ft. 7 in. as a number of feet and a fraction of a foot and multiply by 4.

10. A wire fence 6 wires high is 20 yd. 2 ft. 9 in. long. How many feet of wire are needed for the fence? (Find the result in two ways.)

11. Express decimally to the nearest hundredth of a bushel 4 bu. 3 pk., 2 bu. 1 pk.; 6 bu. 4 qt.; 8 bu. $1\frac{1}{2}$ pk.

12. Multiply the numbers as given in Exercise 11 by 10.

13. Multiply the decimal equivalents of the numbers of Exercise 11 by 10. Which way of multiplying is shorter?

14. Express decimally in gallons 4 gal. 3 qt. 1 pt. and multiply the result by 5.

15. Express decimally in miles 10 mi. 40.5 rd. 12 ft. and multiply by 16.

DIVISION

66. EXAMPLES:

1. How many 5-inch badges can be cut from 5 yd. 2 ft. 11 in. of ribbon?

Solution: (1) 5 yd. 2 ft. 11 in. = 215 in.

(2) $215 \text{ in.} \div 5 \text{ in.} = 43.$

(3) Therefore, 43 badges can be made.

2. Divide 6 bu. 3 pk. 4 qt. by 4.5.

Solution:

(1) $6 \text{ bu.} \div 4.5 = 1 \text{ bu. and } 1.5 \text{ bu. remainder.}$

(2) $1.5 \text{ bu.} + 3 \text{ pk.} = 9 \text{ pk.}$

$$\begin{array}{r} 1 \text{ bu. } 2 \text{ pk. } 1\frac{1}{2} \text{ pt.} \\ 4.5 \overline{) 6 \text{ bu. } 3 \text{ pk. } 4 \text{ qt.}} \end{array}$$

(3) $9 \text{ pk.} \div 4.5 = 2 \text{ pk.}$

(4) $4 \text{ qt. (8 pt.)} \div 4.5 = 1\frac{1}{2} \text{ pt.}$

(5) Therefore, the quotient is 1 bu. 2 pk. $1\frac{1}{2}$ pt.

67. *To divide a compound number by an abstract number, divide the highest denomination by the given divisor; reduce the remainder, if there is one, to the next lower denomination and add it to the units of that denomination. Repeat the division as before.*

If the divisor is a compound number, reduce both divisor and dividend to the same denomination and divide.

WRITTEN EXERCISES

1. Divide by 5: 207 lb. 8 oz.; 2 tons 600 lb.; 5 bu. 2 pk. 4 qt.

Divide :

- | | |
|-----------------------------------|---------------------------------------|
| 2. 12 bu. 2 pk. 2 qt. by 5. | 8. 20 hr. 30 min. 15 sec. by 15. |
| 3. 10 gal. 3 qt. 1 pt. by 1.5. | 9. 4 da. 20 hr. 40 min. by 20. |
| 4. 12 mi. 80 rd. 12 ft. by 16. | 10. 45 bu. 2 pk. 6 qt. by 30. |
| 5. 140 cu. yd. 25 cu. ft. by 18. | 11. 1 T. 500 lb. 12 oz. by 24 lb. |
| 6. 160 sq. ft. 50 sq. in. by 2.5. | 12. 40 yd. 2 ft. 10 in. by 20. |
| 7. 40 tons 1,500 lb. by 200. | 13. 4 gro. 9 doz. 8 units by 6.5 doz. |

Solve :

14. The perimeter of a square is 20 rd. 14 ft. 4 in. What is the length of a side?

15. A rectangular lot with a frontage of 36 yd. 2 ft. 10 in. was divided into 5 equal lots. What was the frontage of each of these lots?

16. A cellar contained 50 cu. yd. 18 cu. ft. of earth, which was drawn away in loads of 1 cu. yd. 3 cu. ft. each. How many loads were there?

17. An elevator contains 5,000 bu. 3 pk. of grain. How many bushels would there be in each car, if this grain were loaded in 8 equal carloads?

18. How many boards 5 yd. 1 ft. long are needed to build a fence 4 boards high and 320 ft. long?

19. Some strips of molding were 1 yd. 1 ft. 3 in. long. How many did it take to go around a room 12 ft. by 15 ft.?

20. A tram car carried 3 cu. yd. and 15 cu. ft. of earth. How many such carloads were there in 9,681 cu. ft. of earth?

21. How many times can a stove's oil tank holding 7 pt. be filled from a 5-gallon can of gasoline?

22. How many bottles containing 1 oz. and 3 dr. can be filled from 77 dr. of medicine?

EQUIVALENTS**Comparison of Liquid Measure and Cubic Measure**

68. A gallon contains 231 cu. in.

WRITTEN EXERCISES

1. One cubic foot of water contains how many cubic inches? How many gallons of water in 1 cu. ft. of water?
2. A tank contains 400 gal. of water. How many cubic feet of water in the tank?
3. An oil car has a capacity of 360 cu. ft. What is its capacity in gallons?
4. How many cubic feet in the capacity of a 5-gallon can?
5. How many barrels ($31\frac{1}{2}$ gal.) of water will a cistern hold that is 8 ft. by 10 ft. by 6 ft.?

Comparison of Dry Measure and Cubic Measure

69. A bushel contains 2,150.42 cu. in.

WRITTEN EXERCISES

1. How many cubic inches in 5 bu.?
2. How many cubic inches in 3 pk.?
3. A bin contained 10,752 cu. in. How many bushels did it hold?
4. How many cubic feet in a bushel measure?
5. A car 32 ft. long, $7\frac{1}{2}$ ft. wide, 8 ft. high, is filled uniformly to $\frac{1}{2}$ of its height with wheat. How many bushels in the car?
6. Using $1\frac{1}{2}$ cu. ft. for a bushel, find the number of bushels in the capacity of a grain elevator 40 ft. long, 40 ft. wide, and 100 ft. high.

7. How many cubic inches in 1 qt. dry measure? How many cubic inches in 1 qt. liquid measure? Which is the larger? By how many inches?

Comparison of Weights

70. Precious metals are weighed by troy weight.

The pound troy contains 5,760 grains, or 12 oz. of 480 grains each. The ordinary pound (*the pound avoirdupois*) contains 7,000 grains.

EXAMPLE: How many pounds avoirdupois does a gold nugget weigh that weighs 39 oz. troy?

Solution : (1) $39 \times 480 \text{ gr.} = 18,720 \text{ gr.}$

(2) $18,720 \div 7,000 = 2\frac{72}{125} = 2\frac{1}{2}$.

(3) Therefore, the nugget weighs $2\frac{1}{2}$ lb.

WRITTEN EXERCISES

1. A bullet weighed 240 gr. troy. What part of an ounce avoirdupois was this?
2. A pound troy is what part of a pound avoirdupois?
3. A carat used to weigh diamonds is 3.168 gr. How many grains in a 3-carat diamond? What part of an ounce avoirdupois does a 3-carat diamond weigh?
4. How many grains in one ounce avoirdupois?
5. An ounce avoirdupois is what part of an ounce troy?

LATITUDE AND LONGITUDE

71. PREPARATORY.

The location of a point on the earth's surface may be indicated by stating its distance north or south of the equator and east or west of a given meridian.

72. Distance in degrees north or south of the equator is called **latitude**.

Latitude 42° N. means 42° north of the equator.

73. Distance measured in degrees east or west of a given meridian is called **longitude**.

The meridian of Greenwich, near London, is usually taken. That of Washington is sometimes used. To say that a place is in longitude 87° W. means that it is 87° west of the meridian of Greenwich.

If you walk directly north, how does your latitude change? Your longitude? If you walk east? West?

74. Angles are measured in **degrees**. 1 degree is divided into 60 equal parts called **minutes**, and 1 minute is divided into 60 equal parts called **seconds**.

75. Degrees, minutes, and seconds are denoted by $^{\circ}$, $'$, and $''$, respectively.

Thus, 12 degrees, 14 minutes, and 17 seconds would be written $12^{\circ} 14' 17''$.

76. EXAMPLES:

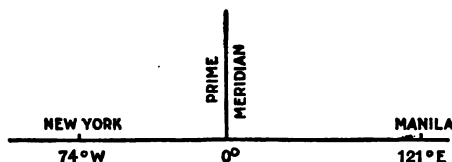
1. Add $30^{\circ} 16' 23''$ and $37^{\circ} 50' 19''$.

30°	$16'$	$23''$	Numbers expressed in degrees, minutes, and seconds are best added by columns, and the usual tests applied to the work of each column. Similarly for subtraction.
37	50	19	
68°	$6'$	$42''$	

2. The longitude of New York is $73^{\circ} 58' 25.5''$ west, and that of Chicago is $87^{\circ} 36' 42''$ west. What is the difference in longitude between these places?

$87^{\circ} 36' 42''$
$73 58 25.5$
<hr/>
$13^{\circ} 38' 16.5''$

3. Find from the figure the difference in longitude between Manila and New York City.



When one place is in east longitude and the other in west, the difference in longitude between the two places is found by adding.

WRITTEN EXERCISES

1. Find the difference in longitude between Berlin and each of the other places in the first column of the table:

PLACE.	LONGITUDE.	PLACE.	LONGITUDE.
Berlin, Germany...	13° 23' 43.5" E.	Manila, Philippines.	120° 58' 00" E.
Dublin, Ireland....	6 20 30 W.	Peking, China.....	116 27 00 E.
Honolulu, Hawaii..	157 51 48 W.	New York.....	73 58 25.5 W.
Paris, France.....	2 20 15 E.	Chicago.....	87 36 42 W.
Tokyo, Japan.....	139 42 30 E.	San Francisco.....	122 25 40.8 W.

2-11. Make and solve 10 other problems from the table.

LONGITUDE AND TIME

77. PREPARATORY.

1. How many hours are required by the earth to make one rotation on its axis?

2. In which direction does the earth rotate?

3. When it is sunrise in New York City, is it before or after sunrise in Boston? In Buffalo? In San Francisco?

4. How many hours elapse while a point of the earth's surface turns through 360°? 180°? 90°? 30°? 15°?

5. When it is noon in Chicago, what time is it in a place 15° west of Chicago? In one 15° east of Chicago?

This table shows the relation between *longitude* and *time*:

TABLE

360° of longitude correspond to 24 hr. of time.

15° of longitude correspond to 1 hr. of time.

1° of longitude corresponds to $\frac{1}{15}$ hr., or 4 min., of time.

15' of longitude correspond to 1 min. of time.

1' of longitude corresponds to $\frac{1}{15}$ min., or 4 sec., of time.

15" of longitude correspond to 1 sec. of time.

1" of longitude corresponds to $\frac{1}{15}$ sec. of time.

78. EXAMPLES:

1. What is the difference in time corresponding to a difference of $200^{\circ} 30' 15''$ of longitude?

Solution: (1) $200 \times 4 \text{ min.} = 800 \text{ min.} = 13 \text{ hr. } 20 \text{ min.}$

(2) $30 \times 4 \text{ sec.} = 120 \text{ sec.} = 2 \text{ min.}$

(3) $15 \times \frac{1}{4} \text{ sec.} = 1 \text{ sec.}$

(4) Therefore, $200^{\circ} 30' 15''$ corresponds to 13 hr. 22 min. 1 sec.

Since a difference of longitude of $200^{\circ} 30' 15''$ is the same as a difference of $159^{\circ} 29' 45''$ taken the other way around the earth, another answer to the problem is 10 hr. 37 min. 59 sec.

2. What is the difference in longitude corresponding to a difference in time of 20 hr. 45 min. 10 sec.?

Solution: (1) $20 \times 15^{\circ} = 300^{\circ}$.

(2) $45 \times 15' = 675' = 11^{\circ} 15'$.

(3) $10 \times 15'' = 150'' = 2' 30''$.

(4) Therefore, 20 hr. 45 min. 10 sec. corresponds to $311^{\circ} 17' 30''$.

Since a difference of more than 12 hr. of time may be reckoned the other way around the earth as a difference of less than 12 hr., this problem has also the answer $48^{\circ} 42' 30''$.

WRITTEN EXERCISES

Express the difference in time that corresponds to each of the following differences in longitude:

1. $10^{\circ} 50' 40''$

4. $14^{\circ} 20' 45''$

7. $25^{\circ} 12' 25''$

2. $5^{\circ} 18' 15''$

5. $90^{\circ} 0' 50''$

8. $17^{\circ} 0' 19''$

3. $8^{\circ} 9' 58''$

6. $12^{\circ} 30' 10''$

9. $180^{\circ} 40' 0''$

Express the difference in longitude that corresponds to each of the following differences in time:

10. 8 hr. 20 min. 15 sec.

13. 10 hr. 0 min. 45 sec.

11. 6 hr. 4 min. 40 sec.

14. 12 hr. 4 min. 10 sec.

12. 5 hr. 54 min. 56 sec.

15. 23 hr. 20 min. 16 sec. (2 Ans.)

Standard Time

79. All places east of a given point have later time than that point; all west of it have earlier time. As a convenience the country has been divided into sections corresponding to



certain meridians, the time of the meridians being taken as the time throughout the sections. This is called **standard time**. The meridians used are 75° , 90° , 105° , 120° west longitude. The map shows the meridians and the corresponding time belts.

ORAL EXERCISES

1. When it is noon, standard time at New York, what is the time at Boston? Denver? Philadelphia? Chicago?
2. San Francisco is how many hours earlier in time than New York?
3. In traveling from New York City to Portland, Oregon, how many times is the watch set back one hour?
4. When it is 12 o'clock midnight at Denver, Colorado, standard time, what is the standard time at New Orleans? New York City? Chicago? Boston? St. Louis?

REVIEW

WRITTEN EXERCISES

Reduce :

1. 16 gal. 2 qt. 1 pt. to pints.
2. .3 mi. + 1000 ft. 10 in. to inches.
3. 25 yd. 2 ft. 9 in. to inches.
4. $4\frac{1}{2}$ sq. yd. + 8 sq. ft. + 100 sq. in. to square inches.
5. 5 cu. ft. 1125 cu. in. to cubic inches.
6. 40,125 oz. to tons, pounds, and ounces.
7. 63,365 in. to miles, feet, and inches.
8. 4,327 cu. ft. to cubic yards and cubic feet.
9. 16 gal. 3 qt. 1 pt. to gallons expressed decimally.
10. 5 bu. 3 pk. 6 qt. 1 pt. to bushels expressed decimally.
11. 5 yd. 2 ft. 6 in. to feet expressed decimally.
12. 6,000 sq. ft. to sq. yd. and sq. ft.

Solve :

13. Write 40 degrees, 30 minutes, and 59 seconds, using the abbreviations. Also 84 degrees, 55 minutes, and 43 seconds.
14. How many seconds are there in 1'? In $2\frac{1}{2}$? In $5\frac{1}{4}$?
15. How many minutes are there in $1\frac{1}{2}^\circ$? In $4\frac{3}{4}^\circ$?
16. How many seconds are there in $6\frac{1}{2}^\circ$? In $2\frac{1}{3}^\circ$?

Add :

- | | |
|---|---|
| 17. 3 wk. 5 da. 20 hr.
<div style="margin-left: 40px;"> 8 4 17
 <hr style="width: 100%;"/> </div> | 20. 12 bu. 3 pk. 7 qt.
<div style="margin-left: 40px;"> 18 2 5
 <hr style="width: 100%;"/> </div> |
| 18. 25 gal. 2 qt. 1 pt.
<div style="margin-left: 40px;"> 18 3
 <hr style="width: 100%;"/> </div> | 21. 40 sq. yd. 8 sq. ft. 100 sq. in.
<div style="margin-left: 40px;"> 50 2 44
 <hr style="width: 100%;"/> </div> |
| 19. $14^\circ 18' 32''$
<div style="margin-left: 40px;"> $26^\circ 49' 52''$
 <hr style="width: 100%;"/> </div> | 22. $27^\circ 4' 50''$
<div style="margin-left: 40px;"> $19^\circ 58' 23''$
 <hr style="width: 100%;"/> </div> |

Subtract :

$$\begin{array}{r} 23. \text{ 10 gal. 1 qt.} \\ \quad \text{7} \quad \text{3} \\ \hline \end{array}$$

$$\begin{array}{r} 24. \text{ 72}^\circ \text{ 18' 12''} \\ \quad \text{43}^\circ \text{ 34' 7''} \\ \hline \end{array}$$

$$\begin{array}{r} 25. \text{ 5 mi. 10 rd. 11 ft.} \\ \quad \text{3} \quad \text{10} \quad \text{13} \\ \hline \end{array}$$

$$\begin{array}{r} 26. \text{ 86}^\circ \text{ 0' 23''} \\ \quad \text{39}^\circ \text{ 27' 48''} \\ \hline \end{array}$$

Multiply :

$$\begin{array}{r} 27. \text{ 3 T. 400 lb.} \\ \quad \text{20} \\ \hline \end{array}$$

$$\begin{array}{r} 28. \text{ 14}^\circ \text{ 36'} \\ \quad \text{9} \\ \hline \end{array}$$

$$\begin{array}{r} 29. \text{ 12 cu. yd. 19 cu. ft.} \\ \quad \text{3.5} \\ \hline \end{array}$$

$$\begin{array}{r} 30. \text{ 8}^\circ \text{ 17' 42''} \\ \quad \text{16} \\ \hline \end{array}$$

Divide :

$$31. \text{ 17} \overline{)420} \text{ yd. 2 ft.}$$

$$32. \text{ 9} \overline{)12^\circ \text{ 36' 9''}}$$

$$33. \text{ 8.2} \overline{)164} \text{ bu. 8.2 qt. 1 pt.}$$

$$34. \text{ 16} \overline{)49^\circ \text{ 17'}}$$

Solve :

35. How many more cubic inches in 10 gal. than in 1 bu.?

36. How many pounds troy make 1 lb. avoirdupois?

37. How many cu. in. more in $3\frac{1}{2}$ bu. than in $3\frac{1}{2}$ cu. ft.?

38. A man sent by express 8 lb. 10 oz. coffee, 4 lb. 14 oz. tea, and 6 lb. 9 oz. spices; the box weighed 3 lb. 11 oz. What was the total weight of the shipment?

39. A lamp holding $1\frac{1}{4}$ qt. is filled 3 times from a 3-gallon can full of oil. How much oil is left in the can?

40. Through an error in manufacturing, a batch of bottles supposed to contain 1 qt., really contained $1\frac{2}{5}$ qt. each. If a milk dealer using these bottles sells a daily average of 650 quart bottles of milk costing him $3\frac{3}{4}$ cts. per quart, how much does he lose in 14 days?

41. Find the difference in longitude between Paris $2^\circ \text{ 20' 15'' E.}$ and New York $73^\circ \text{ 58' 25'' W.}$

42. Find the difference in time between Paris and New York.

43. A bag of wheat weighed 118 lb. 8 oz., another 120 lb. 12 oz., and a third 119 lb. 4 oz. What was the total weight of the 3 bags of wheat?

44. A clerk sold $2\frac{1}{2}$ lb. of paper to one customer, 3 lb. 12 oz. to another, and 4 lb. 4 oz. to another. How many pounds did he sell in all?



45. How many feet of molding, $2\frac{1}{4}$ in. wide, are needed to make a frame for this picture?

46. The sides of a rectangular court measure 32 ft. 8 in. and 64 ft. 9 in. What is the length of the railing surrounding the court?

47. From a ten-gallon can of gasoline a stove holding $7\frac{1}{2}$ qt. is filled. How much gasoline remains in the tank?

48. During four weeks it stormed 1 wk. 2 da. and 20 hr. How many weeks, days, and hours were not stormy?

49. From a 5-gallon can of kerosene were filled a tank holding 1 gal. 1 qt. and 1 pt., a lamp holding 1 qt. and 1 pt., and two other lamps holding $\frac{3}{4}$ of a pint each. How much kerosene was left in the can?

50. From a farm containing 140 A. and 40 sq. rd., a field containing 17 A. and 120 sq. rd. is sold. How many acres and square rods remain in the farm?

51. A street $22\frac{1}{2}$ ft. wide and 486 ft. 9 in. long is to be paved with a layer of pebbles 4 in. thick covered with a layer of crushed stone 2 in. thick. A contractor offers to do the paving at $79\frac{1}{2}$ cts. per cubic yard for the pebbles, $1.83\frac{1}{2}$ cts. per cubic yard for the crushed stone, and $96\frac{3}{4}$ cts. per square yard of surface for all the labor. How much is his bid for the whole?

52. When it is noon on the 90th meridian west, what time is it at 75° W.? At 120° W.? At 105° W.? At 15° E.? At 0°? At 105° E.?

53. The longitude at Chicago is 87° 36' 42" west. When it is noon on the 90° meridian, what is the local time in Chicago? What is the standard time in Chicago at the same instant? What is the difference between local time and standard time in Chicago?

54. The picture shows a plan for irrigating a square farm. The double shaded lines, as N S for example, represent irrigation ditches. Taking each ditch running east and west to be 40 rd. long, how many rods of ditches are there in all?

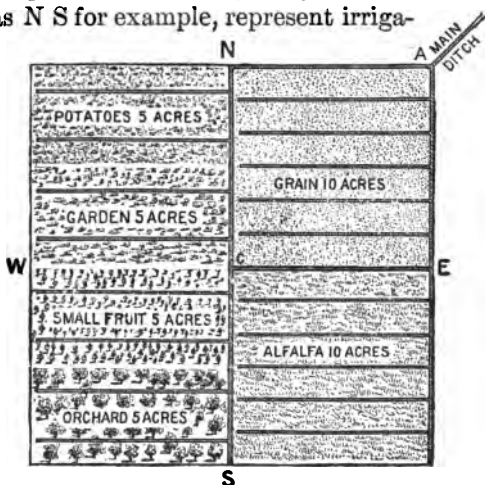
55. Indicate the number of square feet in 1 sq. rd. (1 rd. = 16½ ft.) Find the number of square feet in 1 A.

56. If the ditches on the alfalfa field deliver 2 cu. ft. of water a second, how many hours will it take to deliver a volume of water equal to a lake 6 in. deep over the whole field?

57. If, in excavating the main ditch and the branches for a 10-acre farm, 250 cu. yd. of earth were removed at a cost of 35 cts. a cubic yard, what did the excavating cost?

58. If the surveying cost \$12, the boxes and gates \$125, what was the total cost of the system?

59. What would it cost to irrigate a farm of 25½ A. for one season at \$4.85 per acre?



CHAPTER IV

THE SOLUTION OF PROBLEMS

80. The four essentials in the solution of a problem have already been explained and applied.

- | | |
|-----------------------|--------------------------|
| 1. Read the problem. | 3. Make the calculation. |
| 2. Plan the solution. | 4. Test the work. |

The analysis of a problem is the plan of the solution. It may be expressed orally or written in steps.

EXAMPLE: A dealer buys lamps at \$9 a dozen. For how much must he sell them to gain 50 cts. each?

Plan:

$$(1) \$9 \div 12 = \$(\quad).$$

$$(2) $.75 + $.50 = \$(\quad).$$

Calculation:

$$\begin{array}{r} .75 \\ \cancel{\$9.00} \\ \hline 12 \end{array} + $.50 = \$1.25.$$

Oral Analysis: $\frac{1}{12}$ of \$9, or \$.75, is the cost of 1 lamp. \$.75, the cost, plus \$.50, the gain, is \$1.25, the selling price of the lamp.

WRITTEN EXERCISES

Solve the following problems, writing the analysis in step form and the calculation; also give the oral analysis:

1. $3\frac{1}{2}$ lb. of coffee cost \$1.12. What was the cost of 1 lb.?
2. A dealer paid \$30 for a dozen pairs of shoes; he sold them at a gain of \$.75 a pair. Find the selling price a pair.
3. Taking 500 bunches of lath as a carload, how many carloads would 3,500,000 bunches make?
4. How many $2\frac{1}{2}$ -gallon buckets of water in 5,180 gal.?

81. There is no single plan that will apply to the solution of all problems of arithmetic. But there are two methods widely used, **unitary analysis** and the **equation**.

UNITARY ANALYSIS

82. EXAMPLES:

1. What is the cost of 45 bottles of ink at \$3 per hundred?

Analysis: (1) 100 bottles of ink cost \$3.

(2) Therefore, 1 bottle costs $\frac{1}{100}$ of \$3, or \$.03.

(3) Therefore, 45 bottles cost $45 \times $.03$, or \$1.35.

2. If 14 bu. of peaches make 270 qt. of fruit for canning, how many quarts will 63 bu. make?

Analysis: (1) 14 bu. make 270 qt.

(2) Therefore, 1 bu. makes $\frac{1}{14}$ of 270 qt., or $19\frac{3}{7}$ qt.

(3) Therefore, 63 bu. make $63 \times 19\frac{3}{7}$ qt., or 1,215 qt.

It is usually better first to indicate all the work, then cancel as much as possible, and multiply last of all. The solution of the second example would thus appear:

$$\begin{array}{r} 135 \\ \hline 270 \\ \hline 14 \\ \hline 2 \end{array} \times \begin{array}{r} 9 \\ 63 \\ \hline \end{array} = 1,215.$$

WRITTEN EXERCISES

Analyze and solve as above:

1. If 25 yd. of cloth make 15 vests, how many yards are needed to make 36 vests?

2. If a steamer goes 9 mi. in 48 min., how long will it take to go 51 mi. at the same rate?

3. If the interest on \$330 is \$22, how much is the interest on \$780 for the same time and rate?

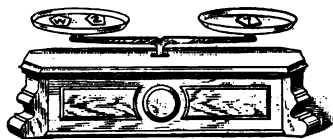
4. If 5 printers can set 100 pages in 1 day, how many pages can 12 printers set in 2 days, at the same rate?

5. If 16 men can dig 980 yd. of sewer trench in 8 days, indicate the amount they can dig in 1 day; the amount 1 man can dig in 1 day; in 20 days; the amount that 28 men can dig in 20 days. Find the last named amount.

THE EQUATION

Principles

83. PREPARATORY.



1. How heavy must the weight, w , be in order that the two pans may balance each other?

The condition of balancing may be represented by $w + 2 = 7$.

State the condition of balancing for each of the following:

2.



3.



4.



84. An **equation** is formed by two equal numbers connected by the sign of equality. The sign separates the equation into two parts called **members**; that on the left is called the **left** (or **first**) member, and that on the right, the **right** (or **second**) member.

85. PREPARATORY.

1. One side of a balance contains $(x + 2)$ oz., and the other an equal weight of $(2 + 8)$ oz. State the equation that expresses that these two scale pans balance. Write it. What is the value of x ?

2. If 2 oz. are taken from each pan, how will the balance be affected? State the equation that expresses the resulting condition.

3. If, instead of 2 oz. being subtracted, 2 oz. had been added to both pans, how would the balance have been affected? State the equation that expresses the resulting condition.

4. If the weight in each pan is doubled, will the balance be affected? If each weight is made $\frac{1}{2}$ as large?

86. *If both members of an equation are increased, diminished, multiplied, or divided by the same number, the resulting members are equal.*

Literal Notation

87. PREPARATORY.

1. If 25 yd. represents the speed of a train per second, what represents its speed per minute? If r represents the speed of a train per second, what represents its speed per minute?

2. If 1000 ft. represents the speed per hour, what represents the speed per minute? If r represents the speed per hour, what represents the speed per minute?

3. If s represents the speed of a train per minute, what represents the distance the train will travel at that rate in 10 min.? In an hour? In 5 hr.? In h hr.?

4. How many seconds will a bicyclist require to ride 3,000 ft. at the rate of 15 ft. a second? At the rate of r ft. a second? How many seconds will he require to ride s ft. at the rate of r ft. a second?

5. If l , b , and h represent respectively the number of feet in the length, breadth, and height of a room, what represents the number of cubic feet in the room?

ORAL EXERCISES

1. When x is the age of one man and y that of another, what does $x + y$ stand for? $x - y$? $2x + y$? $x - 2y$? $\frac{1}{2}x + \frac{1}{2}y$, that is, $\frac{1}{2}$ of $x + y$?

Instead of the expression " $x + y$ stands for" or " n stands for" we may use " $x + y$ is," or " n is."

2. If h is the number of dollars a house costs, and l the number of dollars the lot costs, what is $h + l$? What is $h - l$? What does $h = l$ mean? $h = 2l$? What is the meaning of $h = l + 500$? $h = l - 500$?

3. If l is the length of a rectangle and w is its width, what is $l \times w$? What is $2l + 2b$? What does $l = 2b$ mean? What does $l = w + 20$ mean? $w = l - 10$?

Instead of $l \times w$, we may write lw . In general, absence of a sign between letters denotes multiplication.

State how the second equation is found from the first in each case:

- | | | |
|----------------------|---------------------|-------------------|
| 1. $x + 10 = a + 5.$ | 3. $5x = 17.$ | 5. $2x + 1 = 3.$ |
| $x + 10 - 5 = a.$ | $x = 3\frac{4}{5}.$ | $x = 1.$ |
| 2. $2x + 5 = 20.$ | 4. $4y = 12.$ | 6. $3x + 2 = 11.$ |
| $2x = 15.$ | $y = 3.$ | $x = 3.$ |

Problems Solved by Equations

88. Problems may often be stated most clearly in the form of equations, using some letter to represent the unknown quantity.

WRITTEN EXERCISES

State as equations and find the numbers required:

1. What number increased by 3 equals 12?

Plan: (1) Let n represent the number.

(2) Then $n + 3 = 12.$

(3) Hence, $n = (\quad)$. Why?

2. What number increased by 6 equals 16?

3. What number must be doubled and the result increased by 12 to produce 28?

Plan: (1) Let n stand for the number. (3) Hence, $2n = 16.$ Why?

(2) Then $2n + 12 = 28.$ (4) Hence, $n = (\quad)$.

4. To what number must 16 be added to produce 78?

5. A salesman sold twice as many hats on Tuesday as on Monday, and 6 more on Wednesday than on Tuesday. On Wednesday he sold 16. How many did he sell on Monday?

6. Mary thought of a number; doubled it; added 12 and obtained 58. Of what number did she think?

The equation is $2n + 12 = 58$. Why?

7. In 6 years John will be 4 times as old as James is now. James is now 4 years old. How old is John?

The equation is $x + 6 = 16$. Why?

8. If Mr. Brown's annual salary were increased by $\frac{1}{4}$ of itself, it would lack \$500 of being \$2,000. What is his salary?

9. $\frac{3}{4}$ of the distance from Ann Arbor to Detroit is 30 mi. Find the distance between these places.

Plan: (1) Let x be the distance. (3) $3x = ()$.

(2) Then $\frac{3}{4}x$, or $\frac{3x}{4}$, = 30. (4) $x = ()$.

(5) Test: $\frac{3}{4}$ of 40 = 30, as the problem states.

Find the distances between the cities named in Exercises 10-12, if:

10. $\frac{1}{15}$ of the distance from Detroit to Chicago is 76 mi.

11. $\frac{7}{15}$ of the distance from Boston to Cincinnati is 441 mi.

12. $\frac{2}{3}$ of the distance from Albany to Buffalo is 198 mi.

13. Twice a certain number plus 3 is 15. Find the number.

Plan: (1) Let n represent the number; write 2 times n plus 3 in symbols.

(2) Write the equation that shows that this expression is 15.

(3) What does $2n$ equal? What does n equal?

(4) Test: 2 times the number found plus 3 should be 15.

14. 3 times a certain number plus 6 is 12. Find the number.

15. John is x yr. old, and James is 4 yr. older. The sum of their ages is 26 yr. How old is each boy?

16. If a tennis ball rebounds to $\frac{3}{4}$ of the height from which it was dropped, from what height must it be dropped to rebound $3\frac{1}{2}$ ft.?

17. How high will the same ball rise on the second rebound? On the third?

18. A tower and flag staff are together 100 ft. high; the staff is $\frac{1}{4}$ of the height of the tower. Find the height of each.

19. A house and lot is worth \$3,500; the house is worth 6 times as much as the lot. Find the value of each.

20. The part of a bridge pier under water is $\frac{2}{3}$ as high as the part out of water; the whole height is 45 ft. Find the height of each part.

21. Twice the number of pounds of meat consumed annually per inhabitant in Great Britain is 186% of the consumption per inhabitant in the United States. If the latter is 150 lb., what is the consumption in Great Britain?

22. The number of pounds for the United States, as given in the previous problem, falls 39 lb. short of being three times the number for Germany. Find the latter.

23. An average workman should eat daily a certain weight of starchy foods, 16% of that weight of fats, and 20% of that weight of albuminous foods (protein); the total weight of these three foods consumed daily should be at least $1\frac{1}{4}$ lb. What weight of each is required?

24. 18% by weight of wheat is lost (as bran, etc.) in grinding it into flour. How many 60-pound bushels of wheat are used in making 246 lb. of flour?

25. The weight of bread is 133 $\frac{1}{4}$ % of the weight of the flour used to make it. According to Exercise 24, how many 1-pound loaves of bread can be made from 10 bu. of wheat?

26. In the dairies of the United States the milk of some cows is used to make cheese, that of others to make butter; the average amount of butter produced annually per cow is 130 lb; this is 16 lb. less than twice the average amount of cheese. Find the latter amount.

RATIO AND PROPORTION

89. The quotient of two like numbers is often called their **ratio**.

Thus, the ratio of 4 to 2 is $\frac{4}{2}$, or 2. The ratio of 2 to 4 is $\frac{2}{4}$, or $\frac{1}{2}$.

90. The equality of two ratios is called a **proportion**.

Thus, $\frac{1}{2} = \frac{4}{8}$ and $\frac{2}{3} = \frac{4}{6}$ are proportions.

91. Two or more pairs of numbers are said to be **proportional** when their ratios are equal.

Thus, 2, 6; $1\frac{1}{2}$, $4\frac{1}{2}$; 9, 27 are proportional numbers, because $\frac{2}{6} = \frac{1\frac{1}{2}}{4\frac{1}{2}} = \frac{9}{27}$.

92. Consequently a proportion is an equation, and problems involving proportion can be solved by the use of the equation.

ORAL EXERCISES

1. Compare the ratios: $\frac{1}{2}$ and $\frac{2}{4}$; $\frac{3}{4}$ and $\frac{6}{8}$; and $\frac{1}{3}$ and $\frac{2}{6}$.

2. If $\frac{x}{8} = \frac{1}{2}$, what is the value of x ? If $\frac{x}{3} = \frac{2}{3}$, what is the value of x ? What is x , if $\frac{x}{2} = \frac{3}{2}$?

Find the value of x in each of the following proportions:

3. $\frac{x}{3} = \frac{4}{6}$.

4. $\frac{x}{5} = \frac{12}{10}$.

5. $\frac{x}{1} = \frac{2}{7}$.

6. $\frac{x}{3} = \frac{2}{3}$.

WRITTEN EXERCISES

1. An automobile traveled 70 mi. in 6 hr. How far did it travel in 15 hr. at the same rate?

2. If a sum of money earns \$48 interest in 5 yr., how much will it earn in 16 yr. at the same rate per cent?

3. A city whose population was 40,000 had 4,500 children of school age; the total population increased to 48,000, and the number of children of school age increased proportionally. How many children of school age were there then?

Partitive Proportion

93. The method used to divide a quantity into parts proportional to several given numbers is called **partitive proportion**.

EXAMPLE: An inheritance of \$8,000 was divided among 3 heirs in proportion to their ages: 5 yr., 15 yr., and 20 yr. How many dollars did each receive?

- Solution:* (1) The statement means that each receives the same sum for each year of his age.
(2) Let x = the sum received for 1 year.
(3) Then the first receives $5x$, the second $15x$, and the third $20x$.
(4) Altogether they receive $5x + 15x + 20x = 40x$.
(5) Hence $40x = \$8,000$.
(6) Then, $x = \$200$, $5x = \$1000$, the share of the first, etc.

WRITTEN EXERCISES

1. A invests \$3,000, B \$8,000, and C \$11,000 in a factory. The profits for the year are \$2,000, shared in proportion to the amount of capital invested. What did each receive?

Suggestion: Let p = the profit on \$1000. Then, A receives $3p$, etc.

2. A street was paved at a cost of \$2,400. The cost was assessed on the property owners in proportion to the frontage owned. Mr. A owned 100 ft., Mr. B 300 ft., Mr. C 150 ft., Mr. D 650 ft. How much did each pay?

3. Three men rented an automobile for \$140 per month of 30 days; one used it 1 day each week, another 2 days, and the third 4 days. How much of the monthly rental should each pay?

4. In 100 cu. in. of air there are approximately 21 cu. in. of oxygen and 79 cu. in. of nitrogen; that is, air is composed of 21 volumes of oxygen to 79 volumes of nitrogen. How many cubic inches of each are there in a cubic foot of air?

REVIEW

ORAL EXERCISES

1. When x dollars buys 10 yd. of silk, what does 1 yd. cost? $\frac{1}{4}$ of a yard? $\frac{3}{4}$ of a yard?

2. When c cts. buys 12 oranges, what does 1 orange cost? If d dozen oranges cost a dollars, what is the cost of 1 orange?

3. If a man can do a piece of work in n days, how much of it can he do in 1 day? In 4 days? In d days? How much of it could 3 men do in a days? In x days? How much could q men do in 1 day? In t days?

4. There are a rows of b trees each in an orchard. How many trees are there?

5. A rectangular pile of bricks is a bricks long, b wide, and c high. How many bricks are there in the pile?

6. Represent the length of a rectangle by l in.; its breadth 12 in. less than its length. What represents its perimeter?

7. State and illustrate some processes that may be performed on both members of an equation without destroying the equality.

8. Clara paid c cts. for ribbon and twice as much for silk, spending in all 48 cts. How much did she pay for each article?

When l = length, b = breadth, a = area, and p = perimeter of a rectangle, state the meaning of:

9. $a = lb$.

11. $\frac{a}{l} = b$.

13. $\frac{a}{b} = l$.

10. $p = 2b + 2l$.

12. $p - 2l = 2b$.

14. $\frac{p}{2} - b = l$.

15. What represents the interest on p dollars for 1 yr. at 6%? For 3 yr.? For t yr.?

16. When e is the cost and r is the rate of gain, what represents the actual gain?

17. When x is the rate of motion of a train per hour, y the number of hours traveled, and d the distance, state the meaning of: $d = xy$; $\frac{d}{y} = x$; $2d = 2xy$; $\frac{1}{2}xy = \frac{1}{2}d$.

WRITTEN EXERCISES

1. One box weighed p lb., and another weighed 10 lb. more than the first; the two together weighed 130 lb. What did each weigh?

2. A man 48 years of age is 3 yr. more than 5 times as old as his youngest child. How old is the latter?

3. If Mr. Brown's annual salary were increased by $\frac{1}{3}$, it would lack \$600 of being \$1,800. What is his salary?

4. A certain sum of money was divided into 2 parts in the ratio of 1 to 2; the first part is invested at 6%, the other part at $4\frac{1}{2}\%$; the annual income is \$45. What is the capital?

5. Divide the sum \$5,000 into two parts so that the interest on the larger at 3% a year shall equal that on the smaller at 5% a year.

6. Mr. Gray sold his house and lot for \$17,670; he invested part of the proceeds at $5\frac{1}{2}\%$, and the remainder at $4\frac{1}{2}\%$; the income from each part was the same. How was the sum divided?

7. The sum \$15,000 is invested, part at $4\frac{1}{2}\%$ and the rest at 5%; the annual income is \$687. How is the investment divided?

8. A man bought a tract of land at \$75 an acre. He sold $\frac{1}{3}$ of it at \$84, $\frac{1}{4}$ at \$85, and the remainder at \$90 an acre, thus gaining \$11,280. How many acres were there in the tract? What per cent of the purchase price did he gain?

9. Two persons save \$300 apiece annually; the first had already saved \$1,500 when the second began. In how many years after the second began will he have half as much as the first then has?

10. A bookseller bought 656 copies of a book at a discount of 25%; he sold half of them at list price, and the other half in larger quantities at a discount of 10%; he made \$209.92. Find the list price.

11. The area of the United States exceeds 5 times that of Alaska by 71,180 sq. mi. What is the area of Alaska?

The area of the United States, excluding Alaska and the island possessions, is 3,025,600 sq. mi. Throughout the following problems, "the area of the United States" means this area.

12. 880 times the area of Porto Rico is 2,800 sq. mi. less than that of the United States. Find the area of Porto Rico.

13. If the area of the Philippine Islands were increased by 2,000 sq. mi., 25 times this area would be 99,950 sq. mi. less than the area of the United States. What is the area of the Philippines?

14. The area of all the other island possessions increased by 134 sq. mi. is twice the area of Porto Rico found in Exercise 12. Find the area of these possessions.

15. The area of France increased by 92,946 sq. mi. and the area of the United States when diminished by 2,560 sq. mi. have the ratio 1 to 10. What is the area of France?

16. The area of the United States diminished by 25,600 sq. mi. is $\frac{3}{4}$ of that of China when diminished by 277,170 sq. mi. What is the area of China?

17. The area of the United States, including Alaska and other possessions, is 3,756,884 sq. mi. This area increased by 243,116 sq. mi., is $\frac{1}{3}$ of that of the British Empire when increased by 713,656 sq. mi. Find the area of the British Empire.

18. The area of Germany diminished by 8,830 sq. mi. is $\frac{1}{15}$ of the area of the United States when diminished by 25,600 sq. mi. What is the area of Germany?

19. Three partners divided \$12,000, the gain from one year's business, into parts proportional to the numbers 2, 3, 5. How many dollars did each receive?

CHAPTER V

PERCENTAGE

94. A number per cent means that *number per hundred*.

Thus, "5% of the population are foreign born" means that 5 *persons per hundred* are foreign born.

PERCENTAGE OF NUMBERS

95. The result of applying a rate per cent to a given number is called a **percentage**.

Thus, "5% of 600 bu. of fruit was lost in transit." The rate was 5%, and the percentage 30 bu.

The name *percentage* is applied also to that part of arithmetic which deals with rates per cent.

96. To find a certain percentage of a number, multiply it by the corresponding number of hundredths.

EXAMPLE: 37% by weight of a certain grade of carpet is cotton. How many pounds of cotton in 250 lb. of this carpet?

Solution: 37% of 250 lb., that is, $.37 \times 250$ lb. = 92.5 lb., cotton.

WRITTEN EXERCISES

Find the following percentages:

- | | | |
|------------------------------|-------------------------------|--------------------------------|
| 1. $18\frac{1}{2}\%$ of 435. | 7. 13% of 459. | 13. 54% of 17.9. |
| 2. $27\frac{3}{4}\%$ of 165. | 8. $15\frac{1}{2}\%$ of 960. | 14. 30.6% of .005. |
| 3. 43% of 600. | 9. $3\frac{1}{2}\%$ of 2000. | 15. $19\frac{3}{4}\%$ of 8.06. |
| 4. 69% of 40.8. | 10. 89% of 169. | 16. $9\frac{3}{8}\%$ of 250. |
| 5. 83% of 17. | 11. $14\frac{1}{2}\%$ of 8.5. | 17. 97% of 780. |
| 6. 47.5% of 5. | 12. $12\frac{3}{8}\%$ of 21. | 18. $78\frac{1}{2}\%$ of 2.25. |

Solve:

19. The duty imposed by the United States Government upon Oriental rugs is 42%. How many dollars are collected on each 100 dollars' worth of rugs? How many dollars are collected on a lot worth \$1,575? On a lot worth \$9,763.65?

20. A supply house offered a reduction of 5% on all orders of \$100 or more. What would orders of the following amounts cost the customer:

\$517.98? \$1,563.95? \$850.75? \$1000.50?

21. The timber land of a certain farm was 19% of the whole; the farm contained 168 acres. How many acres were cleared?

97. In solving problems it is often simpler to use the fractional equivalents of rates per cent.

EXAMPLE: The prices in a certain wholesale catalogue are subject to a reduction of $33\frac{1}{3}\%$. What would be the reduction on goods priced at \$690.30 in the catalogue?

Solution: (1) $33\frac{1}{3}\%$ of a number is $.33\frac{1}{3}$, or $\frac{1}{3}$ of the number.

(2) $\frac{1}{3}$ of \$690.30 = \$230.10, the reduction.

Taking $\frac{1}{3}$ of \$690.30 is much simpler than multiplying it by $.33\frac{1}{3}$.

TABLE OF CORRESPONDING NUMBERS

PER CENT.	FRACTION.	PER CENT.	FRACTION.
5	$\frac{1}{20}$	40	$\frac{2}{5}$
10	$\frac{1}{10}$	50	$\frac{1}{2}$
$12\frac{1}{2}$	$\frac{1}{8}$	60	$\frac{3}{5}$
$16\frac{2}{3}$	$\frac{1}{6}$	$66\frac{2}{3}$	$\frac{2}{3}$
20	$\frac{1}{5}$	75	$\frac{3}{4}$
25	$\frac{1}{4}$	80	$\frac{4}{5}$
$33\frac{1}{3}$	$\frac{1}{3}$	$87\frac{1}{2}$	$\frac{7}{8}$
$37\frac{1}{2}$	$\frac{3}{8}$	90	$\frac{9}{10}$

WRITTEN EXERCISES

1. A spelling test paper containing 76 words was marked 75%. How many words were spelled correctly? How many were spelled incorrectly?

2. In an arithmetic test paper containing 25 problems 80% were marked right. How many were right? How many were wrong?

3. The window surface in a schoolroom should be 20% of the floor surface. How many square feet of window surface should a room 20 ft. square have? A room 30 ft. by 50 ft.? One 20 ft. by 25 ft.?

4. 40% of the air in a schoolroom was changed by ventilation in 12 min. At the same rate in how many minutes would it all be changed? The room contained 4,000 cu. ft. How many cubic feet were changed in 12 min.? In 30 min.? In 1 hr.?

5. Find 20% of: 575; 860; \$100.20; \$105.80; 1,580.

6. Find $33\frac{1}{3}\%$ of: 363; 690; \$180.66; \$360.90; \$303.63.

7. Find $16\frac{2}{3}\%$ of: 192; \$246.12; \$252.60; \$126.36; 108.

8. Find $37\frac{1}{2}\%$ of: 160; 360; \$400.80; \$1000.40; \$568.40.

9. Find 75% of: 800; \$760.40; \$1000.80; \$1,204.60; 900.

10. Find $87\frac{1}{2}\%$ of: 160; 360; 400.80; 1000.40; 568.40.

11. Find $66\frac{2}{3}\%$ of: 363; 690; \$180.66; \$360.90; \$303.63.

12. Find 90% of: 810; 465; 863; \$100.20; \$1,005.

13. Find 10% of: 196.3; 1805; 17.25; \$40.90; 1,762.

14. Find $16\frac{2}{3}\%$ of: \$184; \$27.60; 26.90; 4,064; 920.

To Find How Many Per Cent One Number Is of Another

98. *To find how many per cent one number is of another, divide the first by the second and express the quotient in hundredths. The number of hundredths is the number per cent.*

EXAMPLES:

1. 5 is what per cent of 20?

Solution: (1) $\frac{5}{20} = \frac{1}{4} = .25$.

(2) Thus, 5 is .25 of 20, or 25% of 20.

2. 15 is what per cent of 38?

Solution: (1) $15 \div 38 = .39\frac{1}{2}$.

(2) Thus, 15 is $.39\frac{1}{2}$ of 38, or $39\frac{1}{2}$ % of it.

3. 3 is what per cent of 4,500?

Solution: (1) $3 \div 4,500 = .0006\bar{6} = .00\frac{2}{3}$.

(2) Thus, 3 is $\frac{2}{3}$ of 1% of 4,500; that is, $\frac{2}{3}$ % of it.

WRITTEN EXERCISES

- 15 is what per cent of 360? Of 375? Of 420?
- 45 is what per cent of 990? Of 270? Of 360? Of 5?
- 3 is what per cent of 28? Of 190? Of 95?
- Ethel answered correctly 10 questions in a list of 11. What per cent was this? What per cent did she miss?
- Roland solved 11 problems in a list of 13. What per cent did he solve? What per cent did he fail to solve?
- A term of school contained 100 days; a class of 20 pupils had therefore a possible total attendance of 2,000 days; there were total absences amounting to 40 days. What was the rate per cent of absence? Of attendance?
- In a recent month the employees of the Chicago post-office handled 48,000,000 pieces of mail matter; of these 100,000 were addressed defectively. How many pieces per million of mail matter were addressed defectively? How many per thousand? Per hundred? Per cent?
- Of the defectively addressed mail 85,000 addresses were deciphered so as to permit delivery. What per cent were delivered?

To Find a Number From a Given Percentage

99. To find a number of which the percentage at a certain rate is given, divide the percentage by the rate per cent, and multiply the quotient obtained by 100. Or, divide the given number by the fraction corresponding to the rate per cent.

EXAMPLES:

1. 200 is 17% of what number?

Solution by analysis: (1) $200 \div 17 = 11\frac{1}{7}$.

(2) $100 \times 11\frac{1}{7} = 1,176\frac{1}{7}$.

Explanation: Since 200 is 17%, 1 per cent is the quotient of 200 divided by 17, or $11\frac{1}{7}$. Therefore, 100% is $100 \times 11\frac{1}{7}$, or $1,176\frac{1}{7}$.

Use of the equation: (1) Let n be the number sought.

(2) Then, 17% of n , or $.17n$, = 200.

(3) Dividing by $.17$, $n = \frac{200}{.17} = 1,176\frac{1}{7}$.

2. 125 is $66\frac{2}{3}\%$ of a number. What is the number?

Solution: $125 \div 66\frac{2}{3}\% = 125 \div \frac{2}{3} = 125 \times \frac{3}{2} = 187.5$.

WRITTEN EXERCISES

1. An agent received \$25 for the sale of a consignment of goods; this was 5% of the amount of the sale. How many dollars in the amount of the sale?

2. The window space in a room is 20% of the floor space. What is the floor space in a room whose window space is 200 sq. ft.? In one whose window space is 355 sq. ft.?

3. Clarence received a mark of 85% on his spelling examination; he missed 30 words. How many did he spell correctly?

Find the numbers to supply the blanks:

4. 105 is 40% of ().

7. 800 is 75% of ().

5. 400 is $66\frac{2}{3}\%$ of ().

8. 144 is $8\frac{1}{3}\%$ of ().

6. 490 is $87\frac{1}{2}\%$ of ().

9. 279 is 90% of ().

APPLICATIONS OF PERCENTAGE

Gain and Loss

100. Gain or loss in business transactions is commonly expressed by a rate per cent of the cost.

Thus, a merchant bought cloth at \$1 a yard and sold it at \$1.25 a yard, thereby gaining 25%.

WRITTEN EXERCISES

Find the missing numbers :

ARTICLES.	Cost Per Pound.	Selling Price Per Pound.	Part of Cost Gained.	Gain Per Cent of Cost.
1. Nails.....	\$.02	\$.04	()	()
2. Butter20	.26	()	()
3. Sugar04½	.05	()	()
4. Cinnamon90	1.08	()	()
5. Ham.....	.15	()	()	20
6. Cheese	()	.26	()	8½
7. Nuts.....	()	.14	½	()
8. Sausage	()	.12	⅓	()
9. Veal Cutlets ..	()	.20	¼	()

Solve :

10. A bicycle costing \$50 was sold at a gain of 40%. What was the selling price?

11. An automobile was sold for \$650 at a gain of 30%. What was the cost?

If c represents the cost, $1.3c = \$650$ represents the cost plus the gain; that is, the selling price.

Find the cost in each case :

12.	13.	14.	15.
Selling price: \$240	\$390	\$170	\$1,400
Gain or loss: 20% (gain)	30% (gain)	15% (loss)	40% (gain)

Solve :

16. A house costing \$4,000 was sold for \$5,000. What was the rate per cent of gain?

17. An automobile bought for \$3,000 was sold after a year's use for \$1,800. What was the rate of loss?

18. A livery keeper has 10 carriages valued at \$250 each, and 20 valued at \$200 each; at the end of the year he estimates that the value has depreciated 10%. What is the total value at the end of the year?

19. If in the second year the carriages of Exercise 18 depreciate in value by 10% of their value at the beginning of the second year, what is their value at the end of the second year? What per cent of the original value is the total depreciation?

20. If the carriages of Exercise 19 depreciate in value similarly in the third year, what is their value at the end of the third year? What per cent of the original value is the total depreciation to the end of the third year?

21. A man owned two houses costing \$3,000 each; he sold one at 10% gain, the other at 5% loss. How many dollars did he gain in the end? What was the rate per cent of this gain?

22. A man owned three houses costing, respectively, \$2,000, \$3,000, and \$4,000; he sold the first at a gain of 20%, the second at a loss of 10%, and the third at a gain of 5%. How many dollars did he gain in all? What was the rate per cent of gain?

23. A merchant had stock amounting to \$26,000 on hand when he took his annual inventory; during the year he purchased goods costing \$177,000; his next annual inventory showed \$23,000 worth of goods on hand; his accounts showed that during the year \$210,000 were received for goods sold. What was the rate per cent of his gain?

24. A dealer bought a consignment of grape fruit for \$100; he sold 40 cases of it at \$3.50 a case, and the rest, amounting to 46 doz. fruit, at an average of 12 cts. apiece. What was the rate per cent of gain on the whole?

Commission

101. Services in buying or selling for others are often paid by a fixed part of the value of the article sold. Such payment is called **commission**. The rate is usually expressed as a rate per cent.

102. An agent who handles and sells commodities on commission is called a **commission merchant**.

103. A person who sends articles to a commission merchant is called a **shipper, consignor, or principal**.

104. The merchandise sent is called a **consignment**.

WRITTEN EXERCISES

Find the commission at the rate given:

CONSIGNMENT.	RATE.
1. 500 crates of berries sold at \$1.75 a crate	12 $\frac{1}{2}$ %
2. 300 T. of cabbage sold at \$1.25 a ton	8 $\frac{1}{2}$ %
3. 50 crates of eggs at \$5.50 a crate	9 $\frac{1}{2}$ %
4. 200 bbl. of apples sold at \$2.60 a barrel	10%
5. 100 forty-pound tubs of butter sold at 23 cts. a pound ..	5%
6. 48 dressed lambs, 95 lb. each, sold at 10 cts. a pound....	5%
7. 250 lb. of butter sold at 25 cts. a pound.....	4 $\frac{1}{2}$ %
8. 175 baskets of peaches at 85 cts. a basket.....	10 $\frac{1}{2}$ %
9. A commission merchant sold a consignment of peaches for \$235.60, charging 5 $\frac{1}{2}$ %. What was his commission?	
10. Find the commission on 500 lb. of lard sold at 7 cts. a pound, the rate of commission being 3 $\frac{1}{2}$ %.	
11. A commission agent sold 500 baskets of peaches at 75 cts. a basket and remitted \$356.25. What was his commission in dollars? What was his rate of commission?	
12. A commission agent sold some apples on a commission of 15%. His commission amounted to \$63.75. For how much were the apples sold?	

13. A real estate agent sold a house and lot for \$6,000. His commission being $2\frac{1}{4}\%$, what did the owner receive?

14. A piano salesman received a commission of $12\frac{1}{4}\%$ on the amount of his sales. How much did he earn by selling a piano at \$360, and another at \$520?

15. An employment agency charges a commission of 10% on the first month's salary and 5% on the salary of the three succeeding months. How much did a bookkeeper pay the agency for securing him a position at \$70 a month?

16. A commission merchant whose rate of commission was 3% found that after paying \$25 weekly for rent and other expenses, he had earned on an average \$65 a week. What was his average of sales per week?

17. A traveling salesman receives a commission of 5% on the amount of his sales. In a certain week his sales amounted to \$47.18 on Monday, \$94.23 on Tuesday, \$103.14 on Wednesday, \$9.47 on Thursday, \$217.92 on Friday, and \$16.86 on Saturday. How much did he earn that week?

18. Henry sold tickets and received a commission of $7\frac{1}{4}\%$; he sold 16 tickets at \$1 each, 28 at 75 cts. each, 41 at 50 cts. each, and 50 at 25 cts. each. How much did he earn?

19. A real estate agent receives a commission of 5% on the rents he collects; he has 12 houses rented at \$25 a month, 8 at \$30, 2 at \$32.50, 1 at \$35, 4 at \$37.50, and 3 at \$40. How much does he earn per month by collecting these rents?

20. An architect charged a commission of 5% for preparing the plans and supervising the erection of a house costing \$7,580. How much did he receive?

21. An agency for the collection of debts charges a commission of $12\frac{1}{4}\%$ on all sums collected; it collects debts to \$472 for Mr. Williams. How much does Mr. Williams receive?

22. Mr. Reed solicits subscriptions to magazines at a commission of 15%. How much does he earn by securing 5 subscriptions at \$4 each, 2 at \$3, 9 at \$1.50, and 26 at \$1?

Commercial Discounts

105. A reduction made in the price of goods is called a **discount**.

Discounts are made for various reasons, as for large amount purchased, prompt payment, because of damage to the goods, or on special sales to attract custom. Standard prices are fixed for various commodities, and published in *price lists*. When these prices are higher than the usual selling prices, and the selling price is specified by the discount allowed from the *list price*, the price after the discount is deducted is called the *net price*.

ORAL EXERCISES

1. A discount of 10% is offered on the following prices. Find the net prices:

Range.....	\$40	Saw.....	\$3.50
Refrigerator.....	16	Keg of nails.....	3.00
Gasoline stove.....	22	200 ft. of rope.....	13.00
Water heater.....	12	100 lb. of steel rod.....	5.00
Bath tub.....	30	2 dozen door fixtures.....	27.00

2. A firm offers a discount of 20% on the following prices:

FLAGS AND PENNANTS.	SILK— ONE COLOR.	SILK— TWO COLORS.	CASHMERE.
12 in. by 18 in.....	\$0.85	\$1.15	\$0.70
16 in. by 24 in.....	1.00	1.50	0.85
24 in. by 36 in.....	1.35	2.15	1.15

Find the cost of 1 pennant of each kind.

106. When two or more discounts are quoted, the first discount is taken from the list price, the second discount from the amount left after the first discount has been taken off, and so on.

EXAMPLE: If the list price is \$9, and the discounts are 20%, 10%, and 5%, what is the net price?

Plan: (1) $\$9.00 - \$1.80 = \$ (\quad)$. (2) $\$7.20 - \$0.72 = \$ (\quad)$.
(3) $\$6.48 - \$0.32 = \$ (\quad)$, net price.

WRITTEN EXERCISES

1. A bill of goods amounted to \$163 (list price). It was bought at 10%, 10%, and 5% off, and the goods were sold at 10% more than the list price. Find the rate per cent of profit.

2. To what single discounts are the following discounts equivalent: 25% and 10%? $\frac{1}{3}$ and 25%? 40% and 10% and 5%?

3. Compare the net prices of a carriage listed at \$330, sold at the following discounts:

(a) 30%, 20%, and 10%. (c) 20%, 10%, and 30%.

(b) 10%, 30%, and 20%. (d) 10%, 20%, and 30%.

What is the similarity in these cases?

4. A bill for \$480 was discounted regularly at 20%, and a further discount of 2% was allowed for cash payment. What was the amount required to settle the bill?

5. The following table shows prices and discounts:

ARTICLE.	PRICE.	DISCOUNT.
Common plow.....	\$7.50	5%
Sulky-plow.....	19.50	12½%
Hoe.....	0.40	5%
Hay-fork.....	0.60	5%
Truck-wagon.....	70.00	16½%
Runabout.....	55.00	12½%
Surrey.....	90.00	20%
Grain-drill.....	75.00	16½%
Reaper.....	95.00	20%
Mower.....	60.00	15%

How much must a customer pay for each of the following orders, allowing a further discount of 3% for cash:

- (1) 5 Common plows? (4) 3 Truck-wagons? (7) 20 Hay-forks?
 (2) 25 Hoes? (5) 2 Grain-drills? (8) 2 Runabouts?
 (3) 1 Surrey? (6) 2 Sulky-plows? (9) 1 Reaper?

Taxes

107. In towns, counties, and states, the expense of government and the cost of public enterprises are met by collecting from property owners a certain amount on each dollar of property valuation. Sums collected for these purposes are called **taxes**.

108. The **rate** of taxation is expressed by a rate per cent or by an amount on each dollar.

Thus, $\frac{1}{2}\%$, or 5 mills, means a tax of 5 mills on every dollar.

109. Taxes are usually not computed on the full value of the property. The value taken as a basis of taxation is called the **assessed value**.

Thus, if a house and lot worth \$1,600 is valued for taxation at \$1,200, it is said to be *assessed at $\frac{3}{4}$ of its value*.

WRITTEN EXERCISES

1. The following is the annual budget voted by a village board of trustees:

Contingent fund.....	\$10,000	Fourth district, highway fund.....	\$700
Street lighting.....	3,700	Fire department.....	3,500
First district, highway fund.....	1,200	Poor fund.....	1,700
Second district, highway fund.....	1,200	Park fund.....	900
Third district, highway fund.....	1,500	Bridge fund.....	4,000
		Sidewalk fund.....	2,500
		Library fund.....	700

Find the total appropriation.

2. What rate per cent of the total was the contingent fund? The poor fund?

3. Make ten other problems about these appropriations.

4. $\frac{3}{4}$ of the budget mentioned in Exercise 1 was raised by direct taxation, the rate being $1\frac{1}{2}\%$. Find the total assessment on which the tax was based.

REVIEW

ORAL EXERCISES

1. Explain and illustrate the meaning of "number per cent."

2. Explain and illustrate the meaning of "percentage of a number."

3. Find 20% of 80.

4. Find 80% of 20.

5. Which is larger and how much: 60% of 90, or 70% of 80?

6. Find $12\frac{1}{2}\%$ of 72.

7. Find $66\frac{2}{3}\%$ of 750.

8. Which is larger and how much: $87\frac{1}{2}\%$ of 48, or 25% of 120?

9. A railroad 400 mi. long is double-tracked $37\frac{1}{2}\%$ of the way. How many miles are double-tracked? Single-tracked?

10. .05 of a number is what per cent of it?

11. 50% of a number is what part of it? 25%? $33\frac{1}{3}\%$? 20%? $12\frac{1}{2}\%$? $16\frac{2}{3}\%$?

12. How may we find how many per cent one number is of another?

13. 12 is ()% of 18?

19. 3 is ()% of 9?

14. () is 4% of 20?

20. () is 10% of 80?

15. () is $33\frac{1}{3}\%$ of 30?

21. 16 is ()% of 48?

16. 8 is ()% of 40?

22. 16 is ()% of 80?

17. 25 is 200% of ()?

23. 75 is 75% of ()?

18. 20 is ()% of 30?

24. 17 is ()% of 34?

25. 100 lb. is how many per cent of a ton? 1,800 lb. is how many per cent? 500 lb.?

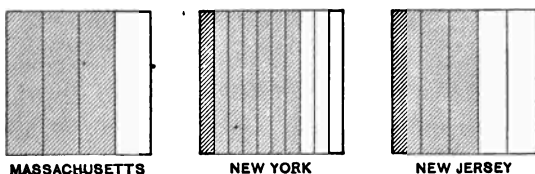
26. 8 oz. is how many per cent of a pound? 4 oz.? 1 oz.? 12 oz.?

27. 18 in. is how many per cent of a yard? 9 in. is how many per cent? 27 in.?

28. 24 qt. is how many per cent of a bushel?

29. A house costing \$2,000 was sold at a gain of 10%. How many dollars were gained? What was the selling price?

30. The squares represent the total population in 1900 of the states named, and the shaded areas represent the proportion of this population living in cities (*urban* population).



How many per cent of the population of Massachusetts lived in cities? In the country?

31. Answer the same question for each of the other states.

32. A real estate agent sold a house and lot for \$10,000; he received 2% of that amount for his services. How many dollars did he earn?

33. What is "commission"? A "commission merchant"? A "consignment"?

34. A commission merchant sold a consignment of goods for \$500; after deducting his commission of 4%, he sent the balance to his principal. How much did the latter receive?

35. At their annual spring clearing sale of men's winter clothing, Taylor and Co. offer a discount of 20% from the marked price of all goods; Mr. Byers purchased articles bearing the following marked prices: one overcoat \$20, one suit \$25, one pair trousers \$5. What did Mr. Byers pay for his entire purchase?

36. What are "taxes"? What is the "assessed value" of property?

37. In a certain state it is provided by law that the total annual taxes on real estate may not exceed 1% of the assessed value of the property. When a man purchases real estate in that state assessed at \$30,000, what does he know about the sum that he may be called on to pay annually as taxes?

38. In a certain state, it is the practice to assess real estate at 60% of its estimated market value. If a house and lot in that state are worth \$1000, what is their assessed value?

39. Under the conditions of Exercise 38, what is the estimated market value of a piece of property assessed at \$1,200?

40. An automobile priced at \$1000 is sold at a discount of 10% and 10%. What was the selling price?

41. An employment agency requires those for whom it secures positions to pay a commission of 10% of the first month's salary. What must the persons pay who secure through the agency the following positions at the monthly salary named: clerk, \$40; stenographer, \$50; bookkeeper, \$65; department manager, \$110; buyer, \$150.

42. Mr. Sellers was very anxious to sell his house, and offered a commission of 5% to any one who would bring him a purchaser. Mr. Trove brought him a man who bought the house for \$8,000. How many dollars did Mr. Trove earn? What did Mr. Sellers actually receive for the house?

43. In a certain store the sales people are paid a fixed weekly salary of \$6 and a commission of 2% on their weekly sales. What would be the total weekly earnings of the following persons making the weekly sales specified: Miss Brown, \$100; Miss Williams, \$300; Miss Rankin, \$150; Miss Allen, \$400?

44. In a community where the rate of taxation was 5 mills, Mr. Andrews paid \$100 in taxes. What was the assessed value of his property? If the rentals on his property were 15% of its assessed value, what were the annual rentals? The assessed value was $66\frac{2}{3}\%$ of the market value. What was the market value? What per cent of the market value were the rentals?

WRITTEN EXERCISES

Find the following :

1. 27% of 862.
 4. 60% of 8,743.
 7. $37\frac{1}{2}\%$ of 72,128.
 2. $33\frac{1}{3}\%$ of 369.
 5. 75% of 4,948.
 8. $16\frac{2}{3}\%$ of 46,170.
 3. 8% of 785.
 6. 63% of 729.
 9. 31% of 49,240.
10. A man's property is distributed as follows: real estate, \$8,000; mortgages, \$5,000; bonds, \$3,000; in bank, \$2,000. What per cent of his total property was invested in each of the forms of investment named?
11. The annual income from the investments named in Exercise 10 was at the following rates: real estate, $7\frac{1}{2}\%$; mortgages, 5%; bonds, $3\frac{1}{4}\%$; bank, 2%. To how many dollars did the man's total income amount?
12. A traveling salesman is offered a position at \$1,800 a year, and another at \$720 a year plus a commission of 15% on the amount of his sales. What would have to be his average monthly sales in order that his earnings in the second position would be equal to his salary in the first?
13. In 1800 the population of the United States was about 5,300,000; of these, $33\frac{1}{2}\%$ were under ten years of age. Find the number of children under ten years of age.
14. In 1900 the population of continental United States was about 76,000,000; of these, 18,000,000 were under ten years of age. What per cent (to nearest .1) of the population were under the age of ten?
15. In 1902 there were about 4,800,000 mi. of telephone wires in the United States; in 1890, there were about 240,000 mi.; and in 1880 about 34,000 mi. Find (to nearest .1) what per cent of the mileage of 1902 was in use in 1890. In 1880.
16. In 1902 there were about 1,300,000 mi. of telegraph wire in use by the commercial telegraph systems of the United States; in 1880, there were about 290,000 mi. Find (to nearest .1) what per cent of the mileage in 1902 was in use in 1880.

17. A property owner's tax is \$30 on an assessment of \$1,500. What is the rate of taxation?

18. The United States silver dollar contains $371\frac{1}{4}$ gr. of pure silver; this is 90% of its weight. What is the weight of a silver dollar?

19. Bees wintered indoors require about $\frac{1}{10}$ as much food as when wintered out of doors. What per cent of the food required out of doors is saved by wintering indoors?

20. According to Exercise 19, if a colony requires 24 lb. of food when wintered indoors, how many pounds would it require when wintered out of doors?

21. A man's tax was \$150 at $1\frac{1}{2}\%$; the assessment was $\frac{2}{3}$ of the actual value of the property. What was its value?

22. A village resident owns a house valued at \$4,500; it is assessed at $\frac{2}{3}$ of its value. What tax does its owner pay at the rate of $1\frac{1}{2}\%$?

23. According to the following table find within $\frac{1}{10}$ of 1% the rate of increase in the total number of votes cast for President of the United States in 1900 over that in 1896:

1896.....	13,923,102	1904.....	13,528,979
1900.....	13,959,653	1908.....	14,852,841

24. As in the last exercise, find to $\frac{1}{10}$ of 1% the rate of increase in 1908 over 1904.

25. Texas is the largest cotton-producing state in the United States; in 1903 the crop was 2,575,000 bales. How many per cent (within 1%) was this of 10,758,000 bales, the total production in the United States?

26. The urban population of Connecticut was 55% of its total population. Represent this relation graphically.

27. Represent graphically the fact that 40% of the total population of the United States lived in cities.

28. The table shows relative prices for a recent period of 14 years:

[The average price for this period is 100.]

YEAR.	Farm products.	Food.	Clothing.	Fuel and lighting.	House furnishing goods.	Lumber, building materials.	Metals and implements.	Drugs and chemicals.	Miscellaneous.
1.....	110.0	112.4	118.5	104.7	111.1	111.8	119.2	110.2	110.8
2.....	121.5	115.7	111.8	102.7	110.2	108.4	111.7	103.6	109.4
3.....	111.7	108.6	109.0	101.1	106.5	102.8	106.0	102.9	106.2
4.....	107.9	110.2	107.2	100.0	104.9	101.9	100.7	100.5	105.9
5.....	95.9	99.8	96.1	92.4	100.1	96.3	90.7	89.8	99.8
6.....	93.3	94.6	92.7	98.1	96.5	94.1	92.0	87.9	94.5
7.....	78.3	83.8	91.8	104.3	94.0	93.4	93.7	92.6	91.4
8.....	85.2	87.7	91.1	96.4	89.8	90.4	86.6	94.4	92.1
9.....	96.1	94.4	93.4	95.4	92.0	95.8	86.4	106.6	92.4
10.....	100.0	98.3	96.7	105.0	95.1	105.8	114.7	111.3	97.7
11.....	109.5	104.2	106.8	120.9	106.1	115.7	120.5	115.7	109.8
12.....	116.9	105.9	101.0	119.5	110.9	116.7	111.9	115.2	107.4
13.....	130.5	111.3	102.0	134.3	112.2	118.8	117.2	114.2	114.1
14.....	118.8	107.1	106.6	149.3	118.0	121.4	117.6	112.6	118.6

What was the rate per cent of decrease in the price of farm products from the first to the tenth year?

29. Answer the same question for the price of food. For the price of lumber. For the price of metals.

30. Make and solve 10 problems about relative prices, similar to Exercises 28 and 29.

31. What is the commission on 3,725 bales of cotton sold at \$12.65 per bale, commission 5%?

32. A bill of books amounting to \$500 was discounted $16\frac{2}{3}\%$ and 5%. What was the net amount of the bill?

33. A property valued at \$80,500 was assessed at $\frac{3}{4}$ of its value. What was the tax at $\frac{4}{5}\%$?

34. When an owner's tax is \$45.15 at the rate of $1\frac{1}{2}\%$, on what valuation is he taxed?

35. A fruit dealer stored 1000 bbl. of apples for 6 mo. at 5 cts. a barrel per month; the storage charge was 15% of the selling price. For how much did the 1000 bbl. sell?

36. A farmer stored 500 tons of cabbage for 3 mo. at 5 cts. a ton per month. The storage charge was 5% of the selling price. For how much did the 500 tons sell?

37. In a recent year the per capita expenditure for bread was \$17.92 and the per capita cost of meat \$9.68. The cost of meat was how many per cent of the cost of bread?

38. As in Exercise 37, the expenditure for clothing was \$19.18. The cost of bread was how many per cent of the cost of clothing?

39. During a recent year 70,900 persons took the Civil Service examinations in the United States, of whom 41% were appointed. How many received appointments?

40. There are 327,000 positions in the Executive Civil Service, 53½% of which are subject to competitive examinations. How many of these positions are subject to examinations?

41. The area of Lake Superior is 32,000 sq. mi. and that of Lake Michigan is 70.2% as large. What is the area of the latter?

42. A school district decides to build a schoolhouse costing \$10,000. The total property assessment of the district is \$2,000,000. What must be the rate of taxation (not counting the cost of collection) in order to raise the \$10,000?

43. There are in the district 115 property owners whose property is assessed as follows:

5 persons, each.....	\$95,000
10 persons, each.....	50,000
25 persons, each.....	10,000
50 persons, each.....	5,000
25 persons, each.....	1,000

Find the tax paid by one person in each class.

44. Find the total tax paid by the persons in each class.

45. The tax collector receives 1% on the amount collected and adds this to the amount collected from each owner. How much, including his commission, did he collect from each of the above owners?

46. A house and lot valued at \$2,400 is assessed at $\frac{3}{4}$ of its value. How many dollars is this? What is the tax at $\frac{3}{4}$ %?

CHAPTER VI

INTEREST AND BANKING

INTEREST

110. A fee paid for the use of money is called **interest**.

111. The rate per cent paid for the use of money is called the **rate of interest**.

112. The money on which interest is paid is called the **principal**.

113. The sum of the principal and the interest for a given time is called the **amount**.

Direct Method

114. *To find the interest, multiply the principal by the rate, and the result by the time expressed in years.*

EXAMPLE: Find the interest on \$650 at 5% for 2 yr. 8 mo.

Solution: \$650 Principal

.05 Rate

\$32.50 Int. for 1 yr. 2 yr. 8 mo. = $2\frac{2}{3}$ yr.

2 $\frac{2}{3}$

\$21.67

65.00

\$86.67 Int. for 2 yr. 8 mo.

115. The solution is given in full to show the method. In practice the computation should be arranged as follows:

$$$.05 \times 650 \times \frac{8}{3} = \frac{\$260.00}{3} = \$86.67.$$

Explanation: The interest on one dollar for one year is 5 cents or \$.05. For \$650, it is 650 times as much as for \$1.

For $\frac{1}{3}$ year, the interest is $\frac{1}{3}$ times the interest for 1 year. The expression $$.05 \times 650$ indicates the interest on \$650 for one year. Such expressions should not usually be worked out separately, but the final result first indicated, as at the bottom of page 109, and then any short process used to abridge the computation. Thus, in the example given, it is easier first to multiply 8 and 5, obtaining 40, then to multiply 650 by 40, then to point off two decimal places in the product (since we used 5 as multiplier instead of .05) and then finally to divide by 3.

116. Cancellation should also be used when possible.

EXAMPLE: Find the interest on \$240 at 6% for 93 days.

$$\text{Solution: } \$.06 \times \frac{2}{\cancel{240}} \times \frac{\frac{31}{93} \cancel{93}}{\cancel{360}} = \$3.72.$$

In ordinary interest the year is regarded as consisting of 12 equal months, or of 360 days.

WRITTEN EXERCISES

Find the interest on the following to nearest cent:

- | | |
|---|---|
| 1. \$500 for 2 yr. 10 mo. @ 5%. | 7. \$800 for 5 yr. 4 mo. @ 6%. |
| 2. \$650 for 3 yr. 6 mo. @ 4%. | 8. \$700 for 3 yr. 9 mo. @ 3%. |
| 3. \$760 for 4 yr. 2 mo. @ 7%. | 9. \$900 for 2 yr. 6 mo. @ 6%. |
| 4. \$500 for 63 da. @ 6%. | 10. \$450 for 30 da. @ 4%. |
| 5. \$350 for 93 da. @ 5%. | 11. \$760 for 63 da. @ $3\frac{1}{4}\%$. |
| 6. \$400 for 120 da. @ $4\frac{1}{2}\%$. | 12. \$325 for 93 da. @ 6%. |

Solve:

13. In a recent year the total debt of New York City was about \$533,000,000. At $3\frac{1}{2}\%$, what was the monthly interest?

14. In the same year, the approximate debts of certain other cities were: Chicago, \$54,000,000; Philadelphia, \$58,000,000; St. Louis, \$24,000,000; and Boston, \$88,000,000. At 4%, what was the monthly interest payment of each city?

15. In the same year, the 174 largest cities of the United States had a total indebtedness of about \$1,426,000,000. At $4\frac{1}{4}\%$, what was the total monthly interest payment of all?

16. A man had \$4,000 invested at $4\frac{1}{2}\%$ interest, and \$2,500 at $5\frac{1}{2}\%$. What was his total income in 3 yr. 4 mo.?

17. A builder borrowed \$2,000 at $4\frac{1}{4}\%$ interest, agreeing to pay back \$1000 and interest at the end of a year, and the rest at the end of another year. How much was the total first payment? The second?

18. A certain university has an endowment fund of \$1,450,000, invested so as to bear interest at the average rate of 3.9% per annum. What is the annual income from its endowment?

19. To meet an emergency, a merchant borrowed \$6,520 for 83 da. at 5% interest. How much interest did he pay?

The Six-Per-Cent Method

117. Taking the year to be 12 months of 30 days each, the *six-per-cent method* is convenient for the following rates: 2%, 3%, 4%, and 6%.

INTEREST AT 6% ON \$1.

Interest on \$1 at 6% for 1 yr. is \$.06. For 1 mo. is .005.

" " " " " 2 mo. is .01. " 1 da. is .000 $\frac{1}{3}$.

EXAMPLE: Find the interest on \$350 at 6% for 2 yr. 4 mo. and 6 da.

Solution :

(1)	Interest on \$1	for 2 yr. at 6%	is $2 \times \$.06$	-	\$.12
(2)	"	" \$1 " 4 mo. " 6%	" $4 \times .005$	-	.02
(3)	"	" \$1 " 6 da. " 6%	" $6 \times .000\frac{1}{3}$	-	.001
(4)	"	" \$1 " 2 yr. 4 mo. 6 da. at 6%		-	.141
(5)	"	" \$350 " " " " "		-	$350 \times \$.141 = \49.35

118. In computing interest at 6%, take 6 cts. per dollar of the principal for each year, $\frac{1}{3}$ of a cent for each month, and $\frac{1}{3}$ of a mill for each day.

WRITTEN EXERCISES

Find the interest on the following at 6% :

- | | |
|-------------------------------------|-------------------------------------|
| 1. \$500 for 2 yr. 8 mo. 12 da. | 15. \$90.85 for 1 yr. 5 mo. |
| 2. \$360 for 4 yr. 9 mo. 10 da. | 16. \$185.60 for 3 yr. 5 mo. 15 da. |
| 3. \$450 for 2 yr. 11 mo. 5 da. | 17. \$400.75 for 5 yr. 6 mo. 20 da. |
| 4. \$700 for 6 yr. 1 mo. 18 da. | 18. \$475.90 for 4 yr. 7 mo. |
| 5. \$1000 for 5 yr. 9 mo. 24 da. | 19. \$790.50 for 3 yr. 18 da. |
| 6. \$2,005 for 10 mo. 12 da. | 20. \$800.25 for 5 yr. 9 mo. 24 da. |
| 7. \$100.50 for 3 yr. 18 da. | 21. \$750.30 for 6 yr. 1 mo. 18 da. |
| 8. \$650.50 for 4 yr. 7 mo. | 22. \$500 for 2 yr. 11 mo. 5 da. |
| 9. \$755.75 for 5 yr. 6 mo. 20 da. | 23. \$5,000 for 4 yr. 9 mo. 10 da. |
| 10. \$900.80 for 3 yr. 5 mo. 15 da. | 24. \$620 for 2 yr. 8 mo. 12 da. |
| 11. \$45,000 for 1 yr. 5 mo. | 25. \$1,800 for 3 yr. 7 mo. 9 da. |
| 12. \$17,000 for 1 yr. 2 mo. | 26. \$14,500 for 3 mo. 18 da. |
| 13. \$832 for 2 yr. 5 mo. | 27. \$2,820 for 1 yr. 24 da. |
| 14. \$63 for 3 yr. 4 mo. | 28. \$36,000 for 4 mo. 16 da. |

119. Interest at 2% may be found by computing the interest on the whole sum for the whole time at 6% and taking $\frac{1}{3}$ of that result.

EXAMPLE: Find the interest on \$350 for 2 yr. 4 mo. 6 da. at 2%.

Solution : (1) The interest at 6% is \$49.35.

(2) $\frac{1}{3}$ of \$49.35 = \$16.45, the interest at 2%.

120. To find interest at 3%, we find the interest at 6% and take $\frac{1}{2}$ of it.

ORAL EXERCISES

1. How can the interest at 4% be found from that at 6%?
2. What must be added to the interest at 6% to find the interest at 7%?
3. What must be added to the interest at 6% to find the interest at 8%?

WRITTEN EXERCISES

Find the interest on the following:

1. \$300 for 4 yr. 9 mo. 10 da. @ 2%.
2. \$500 for 6 yr. 1 mo. 18 da. @ 8%.
3. \$451 for 5 yr. 9 mo. 24 da. @ 4%.
4. \$375 for 3 yr. 18 da. @ 7%.
5. \$400.50 for 4 yr. 7 mo. @ 3%.
6. \$525.50 for 5 yr. 6 mo. 20 da. @ 8%.

Exact Interest

121. The Government and some banks use 365 days as a year and the exact number of days between dates. Interest calculated in this way is called **exact interest**.

EXAMPLE: Find the exact interest on \$150 for 55 da. at 6%.

PLAN: 55 da. = $\frac{55}{365}$ yr. The interest therefore is $\frac{55}{365}$ of $.06 \times \$150$.

122. The following calendar shows the exact number of days between any day of any month and the same day of any other month within a year, leap years excluded:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January..	365	31	59	90	120	151	181	212	243	273	304	334
February..	334	365	28	59	89	120	150	181	112	242	273	303
March ...	306	337	365	31	61	92	122	153	184	214	245	275
April....	275	306	334	365	30	61	91	122	153	183	214	244
May.....	245	276	304	335	365	31	61	92	123	153	184	214
June.....	214	245	273	304	335	365	30	61	92	122	153	183
July.....	184	215	243	274	304	335	365	31	62	92	123	153
August...	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	243	273	303	334	365	30	61	91
October ...	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	96	121	151	182	212	243	274	304	335	365

Thus, from February 17 to July 17 is 150 da. The 150 is found opposite February and under July. Then from February 17 to July 29 is 150 da. + 12 da. = 162 da.

WRITTEN EXERCISES

1. Find the exact interest on \$6,000 from May 5 to August 16 at 6%.

Suggestion: Indicate the interest on \$6,000 for 1 yr. at 6%; then for 1 da.; then for the exact number of days; finally, cancel and reduce.

Find the exact interest on:

2. \$800 from April 15 to June 21 at 5%.

3. \$15,000 from May 29 to June 24 at 4%.

4. \$3,650 from April 2 to May 31 at 5%.

5. \$500 from Jan. 15 to Feb. 23 at 6%.

Interest Tables

123. Bankers and others frequently use an interest table.

A part of an interest table is shown opposite. The complete table consists of a similar page for each dollar from \$1 to \$99, for multiples of \$100, and for multiples of \$1000.

Thus, the interest on \$87 at 3% for 33 da. is found in the column headed 3%, opposite 33 in the central or time column. Result, \$.24:

EXAMPLE: What is the interest on \$87 for 2 yr. 3 mo. 18 da. at 6%?

Solution:

(1)	Interest for 2 yr.	(from table)	\$10.44
(2)	"	" 3 mo.	" "	1.31
(3)	"	" 18 da.	" "26
(4)	"	" 2 yr. 3 mo. 18 da.	\$12.01

ORAL EXERCISES

Find from the table the interest on \$87 for:

1. 27 da. at 12%. 2. 4 yr. at 4%. 3. 11 mo. at 8%.

Solve:

4. A note for \$87 runs 93 da. at 6%. What is the interest?

5. How can interest at $2\frac{1}{2}\%$ be found from interest at 5%?

87 DOLLARS.

3%	4%	5%	6%	TIME.	6%	7%	10%	12%	\$87
.24	.32	.40	.48	33	.64	.56	.80	.96	
.46	.61	.76	.91	63	1.22	1.07	1.52	1.83	
.67	.90	1.12	1.35	93	1.80	1.57	2.25	2.70	
				DAYS.					
.01	.01	.01	.01	1	.02	.02	.02	.03	
.01	.02	.02	.02	2	.04	.03	.05	.06	
.02	.03	.04	.04	3	.06	.05	.07	.09	
.03	.04	.05	.06	4	.08	.07	.10	.12	
.04	.05	.06	.07	5	.10	.08	.12	.15	
.04	.06	.07	.09	6	.12	.10	.15	.17	
.05	.07	.08	.10	7	.14	.12	.17	.20	
.06	.08	.10	.12	8	.15	.14	.19	.23	
.07	.09	.11	.13	9	.17	.15	.22	.26	
.07	.10	.12	.15	10	.19	.17	.24	.29	
.08	.11	.13	.16	11	.21	.19	.27	.32	
.09	.12	.15	.17	12	.23	.20	.29	.35	
.09	.13	.16	.19	13	.25	.22	.31	.38	
.10	.14	.17	.20	14	.27	.24	.34	.41	
.11	.15	.18	.22	15	.29	.25	.36	.44	
.12	.15	.19	.23	16	.31	.27	.39	.46	
.13	.16	.21	.25	17	.33	.29	.41	.49	
.13	.17	.22	.26	18	.35	.30	.44	.52	
.14	.18	.23	.28	19	.37	.32	.46	.55	
.15	.19	.24	.29	20	.39	.34	.48	.58	
.15	.20	.25	.30	21	.41	.36	.51	.61	
.16	.21	.27	.32	22	.43	.37	.53	.64	
.17	.22	.28	.33	23	.44	.39	.56	.67	
.17	.23	.29	.35	24	.46	.41	.58	.70	
.18	.24	.30	.36	25	.48	.42	.60	.73	
.19	.25	.31	.38	26	.50	.44	.63	.75	
.20	.26	.33	.39	27	.52	.46	.65	.78	
.20	.27	.34	.41	28	.54	.47	.68	.81	
.21	.28	.35	.42	29	.56	.49	.70	.84	
.22	.29	.36	.44	30	.58	.51	.73	.87	
				MONTHS.					
.44	.58	.73	.87	2	1.16	1.02	1.45	1.74	
.65	.87	1.09	1.31	3	1.74	1.52	2.18	2.61	
.87	1.16	1.45	1.74	4	2.32	2.03	2.90	3.48	
1.09	1.45	1.81	2.18	5	2.90	2.54	3.63	4.35	
1.31	1.74	2.18	2.61	6	3.48	3.05	4.35	5.22	
1.52	2.03	2.54	3.05	7	4.06	3.55	5.08	6.09	
1.74	2.32	2.90	3.48	8	4.64	4.06	5.80	6.96	
1.96	2.61	3.26	3.92	9	5.22	4.57	6.53	7.83	
2.18	2.90	3.63	4.35	10	5.80	5.08	7.25	8.70	
2.39	3.19	3.99	4.79	11	6.38	5.58	7.98	9.57	
2.61	3.48	4.35	5.22	12	6.96	6.09	8.70	10.44	
				YEARS					
5.22	6.96	8.70	10.44	2	13.92	12.18	17.40	20.88	
7.83	10.44	13.05	15.66	3	20.88	18.27	26.10	31.32	
10.44	13.05	17.40	20.88	4	27.84	24.36	34.80	41.76	
13.05	17.40	21.75	26.10	5	34.80	30.45	43.60	52.20	
3%	4%	5%	6%	TIME.	6%	7%	10%	12%	

WRITTEN EXERCISES

What is the interest on \$87 for :

- | | |
|--------------------------------------|-------------------------------|
| 1. 7 mo. 8 da. at 6%? | 5. 5 mo. 13 da. at 7%? |
| 2. 1 yr. 2 mo. 28 da. at 4%? | 6. 3 yr. 6 da. at 5%? |
| 3. 5 mo. 10 da. at $2\frac{1}{2}$ %? | 7. 5 yr. at $3\frac{1}{2}$ %? |
| 4. 2 yr. 9 mo. at $4\frac{1}{2}$ %? | 8. 10 mo. 23 da. at 2%? |

As we have only one page of the table, the principal must remain \$87 in our problems. With the complete table, interest on any principal is found similarly. To find the interest on \$7,849, the interest would be found (for the given time and rate) on the pages for \$7,000, \$800, \$49, and these amounts would be added.

To Find the Time

124. To find the time when the principal, the rate, and the interest are given,

Divide the given interest by the interest on the principal for 1 yr. at the given rate.

EXAMPLE: The interest on \$300 for a certain time at 6% was \$30. What was the time?

- Solution:* (1) 6% of \$300 = \$18, the interest for 1 yr.
 (2) $\$30 \div \$18 = 1\frac{2}{3}$, the number of years.
 (3) $1\frac{2}{3}$ yr. = 1 yr. 8 mo.

WRITTEN EXERCISES

Fill the blanks in the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
1.	\$500	6%	()	\$45.00
2.	\$300	$5\frac{1}{2}$ %	()	\$67.50
3.	\$250	$4\frac{1}{2}$ %	()	\$33.75
4.	\$400	3%	()	\$64.00
5.	\$475	$2\frac{1}{2}$ %	()	\$35.625
6.	\$600	5%	()	\$100.00
7.	\$900	7%	()	\$84.00

To Find the Rate

125. To find the rate when the principal, the interest, and the time are given,

Divide the interest by the interest on the principal for the given time at 1%.

EXAMPLE: The interest on \$200 for 1 yr. 6 mo. is \$15. What is the rate?

Solution: (1) 1% of \$200 $\times 1\frac{1}{2}$ = \$3, the interest at 1%.

(2) \$15 \div \$3 = 5.

(3) Therefore, the rate of interest is 5%.

WRITTEN EXERCISES

Fill the blanks in the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
1.	\$500	()	2 yr. 6 mo.	\$67.50
2.	\$300	()	90 da.	\$2.25
3.	\$250	()	60 da.	\$2.50
4.	\$400	()	120 da.	\$4.00
5.	\$500	()	1 yr. 3 mo.	\$25.00
6.	\$360	()	3 yr. 8 mo.	\$79.20
7.	\$475	()	2 yr. 9 mo.	\$104.50
8.	\$800	()	4 yr. 6 mo.	\$72.00

To Find the Principal

126. When the rate, time, and interest are given, the principal can be found by

Dividing the interest by the interest on \$1 for the given time and rate.

EXAMPLE: What principal yields \$25 in 2 yr. 6 mo. at 5%?

Solution: (1) 5% of \$1 $\times 2\frac{1}{2}$ = \$.125, the interest on \$1.

(2) \$25 \div .125 = \$200, the principal.

WRITTEN EXERCISES

Fill the blanks in the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
1.	()	6%	60 da.	\$5.00
2.	()	5%	90 da.	\$7.50
3.	()	4½%	120 da.	\$9.00
4.	()	3%	1½ yr.	\$18.00
5.	()	2%	2½ yr.	\$22.50
6.	()	3½%	2 yr. 9 mo.	\$60.75
7.	()	4%	3 yr. 8 mo.	\$36.66½
8.	()	5%	4 yr. 3 mo.	\$31.875

PROMISSORY NOTES

127. When money is borrowed, the borrower usually gives the lender a written acknowledgment of the indebtedness with a promise to pay a certain sum of money with interest at a certain time and place. Such a paper is called a **promissory note**, or simply a **note**.

128. The person who signs the note (agrees to pay) is the **maker**, and the person to whom it is to be paid is the **payee**.

129. The sum loaned is called the **face** of the note, and the face plus the interest is called the **amount**.

130. A note is said to **mature** on the day when it is legally due. When the time is specified in days, exact days are counted in finding maturity; when in months, calendar months are counted. Generally, if a note matures on a holiday or on Sunday, it is payable the day following.

131. Many states still allow **three days of grace** for the payment of notes. Thus, a 30-day note would be legally due in 33 days. When the law allows "days of grace," but does not compel their use, many notes are drawn to read "without grace."

Pay to the order of	\$50.00	Ypsilanti, Mich. February 1, 1902
	Three months	after date without grace I promise to pay
	to the order of	First National Bank, Ypsilanti, Mich.
	Fifty Dollars	
	at the First National Bank Ypsilanti, Mich. Value received with interest at 6% per annum after maturity	
Due		
No.		John Doe

Who is the maker of the note above? The payee? What is its amount? Time? Rate of interest?

WRITTEN EXERCISES

Write notes to cover the following conditions, all to be dated the day of writing:

MAKER.	PAYEE.	FACE.	TIME.	RATE OF INTEREST PER ANNUM.
1. John Smith	Henry Jones	\$125	3 mo.	5%
2. Richard Roe	John Doe	\$87	2 mo.	4%
3. Yourself	Your teacher	\$580	6 mo.	6%

132. If the payee's name only is written in the note, it cannot be sold; but if it reads "Pay to the order of —," or "to bearer," it can be sold, and is called a negotiable note.

133. When a note is transferred, the payee named must first write his signature across the back. This is called **indorsing** the note. Every indorser of a note is responsible for its payment, unless he writes also the words "*without recourse*."

The responsibility of the indorser ceases upon the maturity of the note unless he is promptly notified of its non-payment, but the maker of the note is responsible for its payment until it is paid.

If the payee simply writes his name across the back, he is said to indorse it **in blank**. This makes it payable to the bearer. But if he designates the payee, he is said to indorse it **in full**.

ORAL EXERCISES

1. What is meant by the face of a note?
2. When does a 6-months' note dated April 1 mature without grace? With grace? Are days of grace allowed in your state?
3. A note is drawn by Richard Roe to pay John Doe. Who is the payee?
4. What is meant by a note being negotiable? What must John Doe do to the note mentioned in Exercise 3 in order to make it negotiable?
5. How may a person indorse a note and not become responsible for its payment?
6. What is meant by indorsing a note "in blank"? "In full"?

WRITTEN EXERCISES

1. Write an indorsement in full for a note to your order.
 2. Write an indorsement for a note to your order so as not to become responsible for the payment of the note.
- Use the rate of interest common in your vicinity to find the amounts of the following notes :*
3. Face \$175, date Jan. 10, time 1 yr. 6 mo.
 4. Face \$360, date Apr. 15, time 2 yr. 3 mo. 15 da.
 5. Face \$680, date Feb. 10, time 1 yr. 6 mo. 21 da.
- Solve :*
6. Mr. Brown bought a house for \$1,500. He paid \$700 in cash and \$800 by note at 5%, payable semiannually. What was the amount of interest due at each payment?
 7. Mr. Lee bought a threshing machine for \$300. He paid \$150 in cash and gave his note for the rest with interest at 6%, payable annually. What was the amount of interest due at each payment?

8. A dealer sold an automobile for \$1000, receiving \$400 cash and a note for the rest due in 3 yr. with interest at 6%, payable semiannually. How much interest was paid on the note altogether?

9. A farmer borrowed \$600 on three equal notes, one due in 1 yr., the second in 2 yr., and the third in 3 yr., all with interest at 5%, payable annually. How much was paid at the end of each year, if all payments were made when due? What was the total amount of interest paid?

10. Mr. Perkins bought a house and lot for \$6,000; he paid \$900 cash and gave three notes of equal amounts for the balance, due respectively in 4, 8, and 12 mo. with interest at $4\frac{1}{2}\%$. What sum was due at each payment?

11. Mr. Williams purchased an apartment building for \$12,000. He paid \$4,000 cash and gave five notes of equal amounts for the rest, payable respectively 6, 12, 18, 24, and 30 mo. after date, with interest at 5%, payable semiannually. What was each semiannual payment?

12. A firm accepted the following settlement of an account amounting to \$2,050: A note for \$1000 due in 3 mo., one for \$500 due in 6 mo.; and one for \$550 due in 1 yr., all bearing 6%. What was the amount of these notes? Would the firm have received more or less by accepting a note for \$2,050 for 6 mo. at 6%?

Partial Payments

134. Notes often contain a provision permitting the borrower to pay a part of the principal before the note matures. Such advance payments are usually made on "interest days," the days on which interest is due.

135. When a partial payment of a note is made, the payment is indorsed on the back of the note. This indorsement serves as a receipt.

136. PREPARATORY.

1. A note for \$500 dated Jan. 2, 1909, due in 5 yr. with interest at 6% payable annually, provides the privilege of prepayment in multiples of \$50 on interest days. When is the first interest due? How much is it? If the maker pays \$100 of the face besides the interest, how much does he still owe?

2. What is the interest Jan. 2, 1911? If \$100 of the face is paid on this date, how much does the maker still owe?

WRITTEN EXERCISES

1. A promissory note for \$2,500 due in 4 yr. with interest at 4% payable annually includes the privilege of prepayment. If the maker prepays \$500 and interest each interest day, what is the last payment of principal and interest?

2. On July 1, 1908, John Doe signed a note for \$400 due in 3 yr., interest at 5%, payable semiannually with the privilege of prepayment on interest days. He paid \$150 and interest at each opportunity. What was the interest Jan. 1, 1909? What amount did he still owe? Find the amount still due after each further payment until the obligation was settled.

3. A note for \$500, dated Jan. 2, 1907, with interest at 6%, payable annually for 3 yr. with privilege of prepaying \$100, or more, on interest days, had the following indorsements:

Received, Jan. 2, 1908, \$130. John Doe.

“ “ 2, 1909, 224. John Doe.

Each indorsement included the interest paid on that date. How much was due Jan. 2, 1910?

4. Upon a note for \$350 dated July 1, 1908, with interest at 5%, payable semiannually, the following payments, including interest, were indorsed:

Received, Jan. 1, 1909, \$58.75. Richard Roe.

“ July 1, 1909, 82.50. Richard Roe.

“ Jan. 1, 1910, 105.63. Richard Roe.

What was due July 1, 1910?

In the following notes the interest is payable annually, and prepayment of any sum is permitted on interest dates. Find the amount due at the date of settlement:

	FACE.	RATE.	DATE OF NOTE.	INDORSEMENTS.	DATE OF SETTLEMENT.
5.	\$525	4%	June 15, 1905	June 15, 1906, \$125 June 15, 1907, 250 June 15, 1908, 125	June 15, 1909
6.	250	6	April 1, 1907	April 1, 1908, 120 April 1, 1909, 110	April 1, 1910
7.	1000	5½	Aug. 20, 1906	Aug. 20, 1907, 425 Aug. 20, 1908, 425	Aug. 20, 1909
8.	500	6	May 1, 1905	May 1, 1906, 180 May 1, 1907, 121 May 1, 1908, 115 May 1, 1909, 109	May 1, 1910

INSTALMENT PAYMENTS

WRITTEN EXERCISES

1. Mr. Peters buys a house for \$6,500, on which he pays \$500 cash, and the remainder in monthly payments of \$25, with interest at 6%.

That is, at the end of each month he pays the interest for one month on the sum still due, and besides pays an *instalment* of \$25 on that sum. For example, at the end of the first month he pays \$25 and the interest on the \$6,000 for one month, leaving \$5,975 still due. How much is his first monthly payment? His second? His third? His fourth?

How much has the sum due been reduced in one year? What is still due at the end of the first year?

2. What was his payment at the end of the first month of the second year? The second month?

3. What was his payment at the end of the first month of the third year? The second month? How much has he paid off in ten years? What was his payment at the end of the first month of the eleventh year? The second month?

4. How long would it take him to pay for the house completely? What was his last payment?

5. Make and solve six problems about the accompanying advertisement.

6. Miss Martin bought a piano for \$400 on the installment plan, paying \$40

at the time of purchase, and the remainder in monthly installments of \$10 each, plus the interest on the unpaid balance at 6%. What was her first monthly payment? Her second?

7. What was her payment at the end of the first month of the second year? How long did it take to pay for the piano?

FOR SALE—A BARGAIN

\$7,450

Only \$750 cash and \$80 per month; 4912 Washington Place; interest 4 1-2 per cent; 2-story stone front 8-room residence; quarter-sawed oak finish, steam heat; tile bathroom. Open for inspection.

BANKING

137. Many banks receive deposits and issue check books to the depositor. The check book contains blanks which when filled out are orders upon the bank to pay money and charge the same to the depositor's account.

WRITTEN EXERCISES

1. A man deposited \$580.60 and drew by check \$432.90. How much had he left?

2. A depositor had \$365 to his credit. He deposited a note for \$85 and drew by check \$105.50. How much had he then in the bank?

3. A depositor placed in the bank: \$236.15 and \$906.12; he drew by check \$26.84, \$83.33, \$100, \$11.45, and \$250. How much had he left on deposit?

4. On Jan. 1 a man's bank balance was \$835.16; during January he drew by check \$110.50, \$15.60, \$25.30, \$75.50. What was his balance February 1?

138. In depositing money at a bank, it is customary to file a memorandum, or **deposit slip**.

WRITTEN EXERCISES

1. What was the total amount deposited, according to the deposit slip shown?

2. What was the amount deposited by means of checks? By means of bills and silver?

3. A man entered on a deposit slip gold, \$25; silver, \$123.75; and checks, \$108.50, \$17.50, and \$216.25. How much did he deposit in all?

Find the total amount of deposits made by a merchant on each of the following dates:

DEPOSITED BY <i>No.</i>		
<i>Chas Andrews</i>		
IN THE		
Consolidated National Bank		
OF NEW YORK.		
<i>March 3, 1909.</i>		
	DOLLARS	CENTS
<i>Bills</i>	<i>87</i>	<i>-</i>
<i>Gold</i>		
<i>Silver</i>	<i>325</i>	
<i>Checks. (enter singly)</i>	<i>-</i>	
	<i>4030</i>	
	<i>760</i>	
	<i>137</i>	
	<i>13952</i>	

4.

5.

6.

7.

ITEMS.	NOV. 6.	NOV. 13.	NOV. 20.	NOV. 27.
Gold.....	\$ 50	\$ 50	\$ 20	\$ 10
Silver.....	230	189	300	40
Bills.....	185	264.50	160	300
Checks.....	380.59	740.82	590.94	1,265.17

Certificates of Deposit

139. No interest is paid by banks on money subject to check, unless the daily balance is regularly quite large. Banks will usually pay interest on money not subject to check. In this case a **certificate of deposit** is issued, stating conditions of deposit. If the money is repayable at a fixed time named in the certificate, it is a **time certificate of deposit**. If the money is repayable at any time the holder of the certificate may desire, it is a **demand certificate of deposit**. The amount for which the certificate is drawn is called its **face**.

If the deposit is made for a definite time, the interest is sometimes computed in advance on the sum deposited, and included in the face of the time certificate. The certificate then bears no further interest.

Usually no interest is paid on a demand certificate unless the money is allowed to remain at least a specified time.

John Doe TELLER.	No 147018		Chicago: <u>March 1 1902</u>	
	This Certificate that			
	<u>John Smith</u>		has deposited in this Bank	
	<u>One hundred</u>		<u>Dollars.</u>	
	payable <u>three months</u> after date, to <u>his</u> order, hereon, on			
return of this certificate, with interest at <u>2</u> per cent per annum.				
Amount deposited \$ _____		THE CHICAGO NATIONAL BANK		<u>Richard Roe</u> CASHIER.
months interest \$ _____				
Due _____ 19__ \$ _____				

WRITTEN EXERCISES

1. Is the above a time or a demand certificate of deposit? Find the numbers to fill the blanks in the lower left corner of the certificate.

2. Write a demand certificate of deposit for \$50 on the Chicago National Bank.

The following demand certificates of deposit bear interest at 2% per annum provided the deposit is allowed to remain at least 60 da. Find the amount of interest received in each case :

	FACE.	DATE OF DEPOSIT.	DATE OF PAYMENT.
3.	\$125	Sept. 6, 1909	July 6, 1910
4.	50	Sept. 16, 1909	Oct. 16, 1910
5.	200	Jan. 2, 1909	May 2, 1909
6.	175	July 5, 1910	Sept. 16, 1910
7.	5,000	Aug. 4, 1910	Oct. 14, 1910
8.	20,000	Nov. 17, 1909	Jan. 26, 1910

Solve :

9. On Jan. 5, 1910, Mr. Warren deposited \$600 with the Northern Trust Company, Chicago, receiving in return a demand certificate of deposit stating that interest would be paid at the rate of 2% per annum if left 30 da. On Feb. 17, 1910, he deposited similarly \$1,750. On March 25, 1910, he surrendered both certificates of deposit, receiving in return a check for the total amount of the face and interest of both certificates. Find the amount of the check.

10. Mr. Walker deposited \$150 in the Second National Bank for 4 mo., with interest at $2\frac{1}{4}\%$ per annum. What was the face of his certificate, drawn to include the interest?

Compute the faces of the following certificates of deposit, including interest :

	AMOUNT DEPOSITED.	TIME.	RATE OF INTEREST.	FACE.
11.	\$225	6 mo.	3%	()
12.	120	8 "	$3\frac{1}{4}\%$	()
13.	450	3 "	$2\frac{1}{4}\%$	()
14.	670	2 "	$2\frac{1}{4}\%$	()
15.	1,500	2 "	$2\frac{1}{4}\%$	()

16. Write a demand certificate of deposit with face \$225, bearing 2% interest, and payable to yourself.

Bank Discount

140. If the holder of a promissory note desires to use the money promised before the time of payment, his bank will usually advance him the money, provided he can give satisfactory guarantee that it will be paid when due. This is called **discounting** the note, and the compensation of the bank for this service is called **bank discount**.

Discount, like interest, is computed on the face of the note at a certain rate per cent per annum for the time from the day on which the note is discounted until it is due.

In some localities both days are counted, and the method of exact interest is used.

141. The face of the note less the discount constitutes the sum paid to the holder and is called the **proceeds**.

Thus, if a note for \$100 having 60 da. to run is discounted at 6%, the discount is \$.99, and the proceeds \$99.01.

EXAMPLE: Find the discount and proceeds of a note for \$3,650, without interest, dated Aug. 5, 1908, time 60 da., and discounted Aug. 29 at 6%.

Solution: (1) Aug. 5 + 60 da. = Oct. 4. Aug. 29 to Oct. 4 = 36 da.
 (2) $\frac{3}{100}$ of .06 \times \$3,650 = \$21.90, discount.
 (3) \$3,650 - \$21.90 = \$3,628.10, proceeds.

WRITTEN EXERCISES

In solving the problems below, use 360 da. as 1 yr., and do not include the day of discount.

Find the proceeds:

DATE OF NOTE.	FACE.	TIME.	DATE OF DISCOUNT.	RATE OF DISCOUNT.
1. June 3, 1909	\$1000	60 da.	June 25	6%
2. July 16, 1909	3,500	90 "	Aug. 2	5%
3. May 25, 1910	8,000	60 "	June 28	6%
4. Aug. 4, 1910	1,400	90 "	Aug. 5	5½%
5. Nov. 17, 1910	20,000	30 "	Nov. 17	4½%

Solve :

6. On March 1, a house was sold for \$6,000; the purchaser paid \$1000, gave a mortgage for \$3,000, and his note at 90 da. for the balance; on March 17, holder of the note had it discounted at 6% per annum. How much did he receive?

7. A man held a 6 mo. non-interest bearing note for \$4,000, dated April 1; on April 16 he needed \$2,600 and had to discount the note at 6%; after paying his debt, he deposited the balance on a demand certificate of deposit bearing interest at 2% per annum. How much was the interest on this certificate if he cashed it the day the original note fell due?

142. If a note bears interest, the discount is usually computed upon the amount (the face plus the interest) to the date of maturity. The method varies.

EXAMPLE: Find the discount at 6% on a note of \$500 with interest at 6% for 3 mo., discounted 1 mo. after date.

Solution: (1) $\$500 + \frac{1}{4} \times .06 \times \$500 = \$507.50$, the amount.

(2) $\frac{1}{4} \times .06 \times \$507.50 = \$5.08$, the discount.

(3) $\$507.50 - \$5.08 = \$502.42$, the proceeds.

WRITTEN EXERCISES

1. If the notes in Exercises 1, 2, 3 on page 128 bear interest at 5%, find the proceeds.

Find the discounts and proceeds of the following notes :

	DATE OF NOTE.	FACE.	TIME.	DATE OF DISC.	RATE OF INT. DISC.	
2.	Jan. 1, 1908	\$250	30 da.	Jan. 15, 1908	6%	5%
3.	Feb. 15, 1908	350	60 "	Mch. 15, 1908	5½	5
4.	Dec. 20, 1907	400	90 "	Feb. 1, 1908	4½	4
5.	Jan. 1, 1909	425	120 "	Mch. 10, 1909	6	5
6.	June 1, 1908	600	1 yr.	Jan. 1, 1909	4	3½
7.	April 1, 1909	500	1½ "	Apr. 1, 1910	4½	4

COMPOUND INTEREST

143. Interest that is computed on the original principal only is called **simple interest**.

144. When the conditions of payment are that interest due but not paid shall be added to the principal and bear interest thereafter, the total interest earned is called **compound interest**.

An instance of the payment of compound interest is found in savings banks. It is the custom of such banks to compute interest on their deposits at stated intervals and to add to each deposit the interest it has earned, thus paying interest upon interest.

When the interest is added to the principal semiannually, it is said to be *compounded semiannually*; when added annually, it is said to be *compounded annually*.

Compound interest is almost invariably computed by the use of tables that usually give the result of compounding annually the interest on 1 dollar at various rates. The interest is found by deducting the principal from the given amount.

A small portion of a compound interest table is given below:

AMOUNT OF \$1 AT COMPOUND INTEREST, COMPOUNDED ANNUALLY:

YEARS.	RATE: 2%	2½%	3%	4%	5%
1	1.02000	1.02500	1.03000	1.04000	1.05000
2	1.04040	1.05063	1.06090	1.08160	1.10250
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15927	1.21665	1.27628
6	1.12616	1.15969	1.19405	1.26532	1.34010
7	1.14869	1.18869	1.22987	1.31593	1.40710
8	1.17166	1.21840	1.26677	1.36857	1.47746
9	1.19509	1.24886	1.30477	1.42331	1.55132
10	1.21899	1.28008	1.34392	1.48024	1.62889
11	1.24337	1.31249	1.38423	1.53945	1.71033

Thus, \$1 at 3% compounded for 4 yr. amounts to \$1.12551. This is found in the column under 3% and in the line opposite 4 at the left. The compound interest on \$100 for 4 yr. at 3% would be $100 \times \$1.12551$, and similarly for any other principal.

Interest otherwise compounded can easily be found from the tables. Thus, interest at 4% per annum compounded semiannually is the same as interest at 2% compounded annually for twice the number of years.

WRITTEN EXERCISES

Find from the table the amounts of the following, compounded annually:

	PRINCIPAL.	TIME.	RATE.
1.	\$1000	5 yr.	3%
2.	500	3 "	4
3.	650	4 "	2
4.	425	3 "	2½
5.	775	4 "	2
6.	800	8 "	4
7.	5,000	10 "	5
8.	6,800	12 "	2
9.	350	11 "	2½
10.	832	7 "	3
11.	326	9 "	4
12.	1,500	6 "	5
13.	2,000	8 "	2
14.	2,000	8 "	4

Solve:

15. A man deposited \$2,000 in a savings bank, July 1, at 3% per annum, interest credited semiannually. What interest was credited the following Jan. 1? What was the new principal?

16. What was the interest and new principal the next July 1? How much money would the man have in the bank at the end of 3 yr.?

REVIEW**ORAL EXERCISES**

1. What is meant by a promissory note? By the maker of a note? The payee?
2. How would you word a promissory note for \$200 dated to-day and payable in 3 yr. with interest at 5%?
3. What is meant by days of grace? Are days of grace legal in your state?
4. What is meant by negotiable? What is indorsing a note? Illustrate an indorsement in blank. In full.
5. How is exact interest computed? What do business men commonly use in computing interest?
6. What is a certificate of deposit? A demand certificate? A time certificate?
7. How is the interest computed on deposits made in a savings bank?
8. What is bank discount? Illustrate your answer.
9. What is meant by proceeds? Illustrate your answer.
10. How does a partial payment note differ from an ordinary promissory note? How is the interest computed on such a note?
11. What is the interest at 5% on a promissory note for \$500 for 6 mo.?
12. What is the amount of the promissory note of Exercise 11?
13. When does a 6 mo. note mature, dated Jan. 2, 1909, if drawn in a state where grace is allowed?
14. How may one indorse a note and not become responsible for its payment?
15. What is the bank discount at 6% on a note for \$200 due in 3 mo.?

WRITTEN EXERCISES

Find the interest at 6% on the following notes :

1. \$675.50 for 3 yr. 7 mo.
2. \$1,007.75 for 4 yr. 5 mo.
3. \$380.25 for 2 yr. 10 mo. 15 da.
4. \$160 for 5 yr. 4 mo. 10 da.
5. \$7,000 for 2 mo.
6. \$60,000 for 1 mo.
7. \$463,000 for 5 mo. 10 da.

Find the interest on :

8. \$560 for 2 yr. 6 mo. 18 da. at 2%.
9. \$840.90 for 3 yr. 11 mo. 10 da. at 3%.
10. \$690 for 5 yr. 2 mo. 20 da. at $3\frac{1}{2}\%$.
11. \$1,200 for 4 yr. 3 mo. 27 da. at $4\frac{1}{2}\%$.
12. \$230 for 1 yr. 4 mo. at 5%.
13. \$4,750 for 6 mo. at $3\frac{1}{2}\%$.
14. \$3,000 for 5 mo. 13 da. at $4\frac{1}{2}\%$.

Find the exact interest at 5% on :

15. \$350 from May 1, 1909, to Feb. 15, 1910.
16. \$275.75 from Dec. 14, 1910, to May 1, 1911.
17. \$43,000 from Nov. 10 to Dec. 31, 1910.
18. \$6,500,000 from Mar. 5 to Mar. 23, 1911.

Solve:

19. \$400 put out at interest at $3\frac{1}{2}\%$ earned \$21 in a certain time. How many years was it lent?
20. \$350 was lent for 2 yr. 3 mo. and earned \$27.56. What was the rate of interest?
21. A man gave a 4 months' note at 6% for \$300. How much must he save per day to accumulate sufficient money to pay principal and interest of the note when due?

Fill the blanks of the table:

	PRINCIPAL.	RATE.	TIME.	INTEREST.
22.	\$1000	6%	1 yr. 2 mo.	\$ ()
23.	3,000	()	1 yr. 4 mo.	200
24.	()	4	6 mo.	120
25.	400	$2\frac{1}{2}$	()	3
26.	()	$3\frac{1}{4}$	8 mo. 15 da.	330
27.	8,500	$5\frac{1}{2}$	9 mo. 21 da.	()
28.	320	4	()	80
29.	2,000	()	2 mo. 20 da.	20
30.	570	$4\frac{1}{2}$	23 da.	()
31.	()	6	18 da.	540
32.	200,000	$4\frac{1}{4}$	1 mo. 10 da.	()
33.	50,000	3	()	25

Solve:

34. A certain sum of money earned \$45.84 at $5\frac{1}{2}$ % in 1 yr. 8 mo. What was the principal lent?

35. A man bought a house and lot for \$8,000 and paid \$1,500 in cash; he gave a note for the rest for 4 yr. 6 mo. at $5\frac{1}{2}$ %. What was the total interest due on the note?

36. A bank has a capital of \$150,000 and a surplus (undivided profits) equal to 101.32% of this. Find the surplus.

37. Find the surplus in each case as in Exercise 36:

	(a)	(b)	(c)	(d)	(e)
Capital.....	\$250,000	\$500,000	\$300,000	\$750,000	\$100,000
Surplus.....	210.73%	134.82%	120.54%	119.73%	120.22%

38. At $4\frac{1}{2}$ % how long would it take \$1,800 to earn \$100?

39. How much more does \$1000 earn in 12 yr. at 5% compound interest, than at the same rate of simple interest?

40. A man paid off \$1000 in monthly installments of \$100 each plus the interest on the unpaid balance at the rate of 6% per annum. What was his first payment? His second?

41. Make and solve five problems about the annexed advertisement, using $5\frac{1}{2}\%$ as the rate of interest.

BE YOUR OWN LANDLORD

6405 JACKSON AVE.

For Sale: New detached 2-story brick and stone residence, containing parlor, dining-room, kitchen, 4 bedrooms and bath; an ideal home. Price \$5,500. ONLY \$500 CASH and monthly payments of \$40 and interest required to buy this house.

42. Find the interest upon the following certificates of deposit at 2% , allowing 30 da. from date of deposit before the certificate draws interest:

FACE.	DATE OF DEPOSIT.	DATE OF PAYMENT.
(a) \$235	May 2, 1912	March 15, 1913
(b) 46.50	Apr. 1, 1911	Dec. 20, 1911
(c) 186.25	June 15, 1910	July 10, 1911

43. Find the proceeds of the following notes:

DATE OF NOTE.	FACE.	TIME.	DATE OF DISCOUNT.	RATE OF DISCOUNT.
(a) June 20, 1909	\$250	90 da.	July 25	$5\frac{1}{2}\%$
(b) July 15, 1910	375	60 da.	Aug. 10	6
(c) Feb. 1, 1911	1,950	90 da.	Apr. 25	5

44. A note for \$675 bearing 6% interest, dated Aug. 1, 1908, and due in 5 yr., had the following indorsements:

Jan. 1, 1909, \$200.

June 1, 1911, \$100.

Jan. 1, 1910, \$150.

July 1, 1911, \$75.

No interest having been paid, find the amount due at maturity.

45. What is the compound interest on \$100 for 4 yr. at 3% , compounded annually?

46. What is the amount of \$500 for 3 yr. at 4% , compounded annually?

47. What is the compound interest on \$250 for 2 yr. at 2% , compounded semiannually?

48. At simple interest at 7% , how many years will it take \$150 to earn \$39.375?

49. A man bought a house for \$10,000, paying \$2,000 cash and \$1000 annually with interest at $5\frac{1}{2}\%$. How much did he pay for the house in principal and interest?

CHAPTER VII
BUSINESS APPLICATIONS
ORDERING GOODS
WRITTEN EXERCISES

1. Make out a bill for the goods mentioned in this order:

18 Main Street, Rockwood, N. J. June 8, 1909
Mr. John Marshall, New York City.
Dear Sir :
Please send me by express :
1 set Dickens' complete works, advertised at \$18. 1 3-foot wire bed-spring, Cat. No. 872, \$4.50 1 sable fur boa, advertised at \$7.95
and charge to my account.
Yours truly, <i>John Perkins</i>

Write an order and a bill for each of the following, the purchases being made of Wiley Smith & Co., Chicago :

2. Roland Ames, Springfield, Missouri:
- | | |
|--|---------|
| 2 lawn mowers, catalogue number 6900, each | \$2.33. |
| 2 doz. camp chairs, " " 7053, " | .70. |
| ½ doz. folding cots, " " 7770, " | 1.65. |

3. A. R. Childs, Newark, New Jersey:

Two 5-pound cans Java coffee, at $31\frac{1}{2}$ cts. a pound.

5 bbl. flour, catalogue number A 723, at \$5.50 a barrel.

3 doz. cans sardines, catalogue No. A 271, at \$4.25 a dozen.

RECORD OF SALES

145. A common method of recording sales is that of noting in a book, called a **day book**, the name of the customer, a description of the goods bought, and whether the price is charged to the customer's account or paid in cash.

WRITTEN EXERCISES

1. The following is a coal dealer's record of one day's sales. Verify the sums in each column, and find the total amount of the sales.

<i>February 18, 1909.</i>		<i>Account Cash</i>			
<i>Thomas Carlton</i>	<i>1/2 Cord wood</i>				<i>1 25</i>
<i>Ray Rankin</i>	<i>Nut Coal 304 lb</i>				<i>88</i>
<i>Arthur Ailey</i>	<i>Smith . 170 .</i>				<i>50</i>
<i>James O'Brien</i>	<i>Stove . 80 .</i>				<i>25</i>
<i>Matthew Spencer</i>	<i>Nut . 242 .</i>				<i>6 17</i>
<i>M. C. Carr</i>	<i>Stove . 400 .</i>		<i>8 00</i>		
<i>Norval Johnson</i>	<i>Nut . 80 .</i>				<i>25</i>
<i>Peter Mason</i>	<i>Nut . 800 .</i>		<i>24 00</i>		
<i>H. H. Goodrich</i>	<i>Nut . 625 .</i>				<i>1 64</i>
<i>Myron Cobb</i>	<i>Stove . 260 .</i>		<i>5 93</i>		
<i>Edw. Case</i>	<i>Nut . 400 .</i>				<i>12 00</i>
<i>Hubert Nixon</i>	<i>1/2 Cord Hard</i>		<i>1 25</i>		
	<i>Totals</i>		<i>39 18</i>	<i>22 94</i>	

2. On a certain day a dealer's cash items were: \$2.50, \$5.75, \$11.16, \$4.93, \$.30, \$1.15, \$.33, \$4.60; the account items were: \$4.78, \$13.60, \$5.12, \$3.17. Find the total.

3. Make other records and find the totals.

BILLS AND RECEIPTS

146. When a bill is paid it is customary to give a written acknowledgment of payment, called a receipt. The receipt may be either written or stamped on the bill itself, as in the bill shown below, or it may be drawn up separately.

APPROX ALL CORRESPONDENCE TO D. APPLETON & COMPANY, NEW YORK
*ADDRESS ALL PACKAGES TO EMST AVE. AND NEWTON ST., BROOKLYN, N. Y.

No. _____

NEW YORK Mar 1 1909

D. APPLETON & COMPANY
PUBLISHERS AND BOOKSELLERS

SOLD TO Mr. Frank Morris
Cleveland, Ohio

TERMS _____

NOT RESPONSIBLE FOR GOODS LOST OR DAMAGED IN TRANSIT, SHIPPED OR RECEIVED FOR DELIVERY, OR SENT BY MAIL.
CLAIMS FOR ALLOWANCES MUST BE MADE UPON RECEIPT OF GOODS

CONVEYANCE _____

		EDUCATIONAL	WHOLESALE	TOTAL
1	Young & Jackson's Arithmetic Book	30		
1	2	40		
1	High School Science in English	60		
1	Warner Culture Primer	30		
				165

D. APPLETON & CO
MAR 1 1909

WRITTEN EXERCISES

Make out receipted bills for the following items, with yourself as purchaser, and as seller some firm that deals in the commodities named:

- 3 sacks of flour at \$1.69, one 16-pound ham at 12½ cts.
9 lb. of tea at 39 cts. a pound.

2. 1 dining table, \$27; 8 chairs at \$2.98 each.

3. 6 violin strings at 19 cts. each; 1 cake rosin, 8 cts.; 3 copies of sheet music at 29 cts. each, 5 at 15 cts., and 1 at 60 cts.

Write receipts showing that you have made the following payments:

4. A monthly instalment of \$10 for the current month on a piano that you have bought from a dealer known to you.

5. To the publisher of a local paper or magazine for one year's subscription in advance. (Use an actual name and rate.)

Make a receipted bill for each of the following:

6. 12 Geographies at \$1 each, 125 Spellers at 22 cts. each, 300 Primers at 25 cts. each, 50 Arithmetics at 35 cts. each. Bought by A. Dunn from Baker, Taylor and Co.

7. E. D. Clark bought of R. H. Macy and Co.

40 yd. of silk at	\$.90 a yd.
120 dress patterns at	5.25 each
2 doz. blankets at	3.75 per pair
13 mattresses at	4.65 each

8. F. Klein purchased of Tiffany and Co.

5 gold watches at	\$75.50 each
12 sets of silver knives at	48.00 a set
22 sets of silver spoons at	22.50 a set
5 silver trays at	27.50 each

9-13. Make five bills and receipt them, using the following prices:

Men's hats	\$3.50	Calico	\$.06 a yd.
" shoes	4.00	Muslin08 "
" hose75	Bleached muslin12 "
Silk vests	2.25	Broadcloth	1.75 "
Collars	1.50 per dozen.	Lansdowne	1.25 "
Ties50 each	Cheviot75 "

CASH ACCOUNTS

147. An account of money received and paid out is called a **cash account**.

Dr. stands for debtor and *Cr.* for creditor. "Cash" may be thought of as a debtor to the person keeping the account for all money received and as creditor for all money paid out.

<i>Cash Account of M. P. C.</i>					
				<i>Dr.</i>	<i>Cr.</i>
<i>Sept.</i>	<i>1</i>	<i>Cash on hand</i>		<i>16.15</i>	
	<i>1</i>	<i>Wage</i>		<i>13.50</i>	
	<i>2</i>	<i>Rent</i>			<i>14.00</i>
	<i>3</i>	<i>Heat</i>			<i>15.00</i>
	<i>5</i>	<i>School District</i>			<i>3.62</i>
	<i>6</i>	<i>Insurance</i>			<i>5.07</i>
	<i>7</i>	<i>Milk Tickets</i>			<i>1.00</i>
		<i>Balance</i>			<i>4.26</i>
				<i>27.65</i>	<i>27.65</i>
<i>Sept.</i>	<i>1</i>	<i>Cash on hand</i>		<i>4.26</i>	

WRITTEN EXERCISES

Make out and balance the following accounts:

1. Cash on hand Jan. 1, \$500. Expenditures: Jan. 5, \$80.30; Jan. 10, \$30.75; Jan. 15, \$4.25; Jan. 21, \$106; Jan. 30, \$40.80.

2. Cash on hand Monday, \$90.50. Receipts: \$10 a day for the week of 6 days. Expenditures: Monday, \$7.50; Tuesday, \$12.25; Thursday, \$18.40; Saturday, \$21.43.

3. Cash on hand July 1, \$74.90. Receipts: July 6, \$60.70; July 15, \$50; July 21, \$18.90; July 31, \$50. Expenditures: July 2, \$25.50; July 7, \$6.50; July 16, \$40.59.

4. Cash on hand Oct. 1, \$185.50. Expenditures: Oct. 3, \$16.50; Oct. 11, \$89.70; Oct. 15, \$5.53; Oct. 25, \$17.75; Oct. 28, \$15.90; Oct. 30, \$12.

5. Jan. 1, '09, on hand \$75. Expenditures: Jan. 2, books \$3.50; Jan. 10, board \$7.50; Jan. 15, rent \$25; Jan. 25, clothes \$22.50. Jan. 30, salary received, \$250.

Balance the following accounts :

6.

7.

EXPENDI- TURES.	CASH.	EXPENDI- TURES.	CASH.
May 1, \$10.	On hand \$125.50	July 1, \$125.50	On hand \$1000
May 3 15.75		July 5 75.60	Profits 500
May 5 8.30		July 10 200.30	
May 10 12.60	Int. 20.00	July 16 190.05	Int. 300
May 15 1.25		July 25 400.25	Dividends 650
May 25 50.00	Salary 175.00	July 30 90.83	Profits 2010

REMITTANCES

Registered Mail

148. Money may be safely forwarded by mail in a letter that is **registered** at a post-office. The fee for this is 8 cents. A receipt showing the delivery of the letter is returned to the sender by the post-office.

Postal Money Orders

149. Payment to a person at a distance may be made also by sending him a **postal money order** purchased at a post-office. This is an order directing the postmaster at the place where the other person lives to pay him the money. The person presenting the order for payment must be known to the postmaster or be identified.

CASH ACCOUNTS

147. An account of money received and paid out is called a cash account.

Dr. stands for debtor and *Cr.* for creditor. "Cash" may be thought of as a debtor to the person keeping the account for all money received and as creditor for all money paid out.

<i>Cash Account of M. F. C.</i>					
			<i>Dr.</i>	<i>Cr.</i>	
<i>Sept.</i>	<i>1</i>	<i>Cash on hand</i>	<i>16</i>	<i>15</i>	
	<i>1</i>	<i>Wages</i>	<i>13</i>	<i>50</i>	
	<i>2</i>	<i>Rent</i>			<i>14.00</i>
	<i>3</i>	<i>Heat</i>			<i>1.50</i>
	<i>5</i>	<i>School Books</i>			<i>3.62</i>
	<i>6</i>	<i>Insurance</i>			<i>5.07</i>
	<i>7</i>	<i>Milk - Truck</i>			<i>1.00</i>
		<i>Balance</i>			<i>4.26</i>
			<i>29.65</i>	<i>29.65</i>	
<i>Sept.</i>	<i>1</i>	<i>Cash on hand</i>	<i>4.26</i>		

WRITTEN EXERCISES

Make out and balance the following accounts:

1. Cash on hand Jan. 1, \$500. Expenditures: Jan. 5, \$80.30; Jan. 10, \$30.75; Jan. 15, \$4.25; Jan. 21, \$106; Jan. 30, \$40.80.

2. Cash on hand Monday, \$90.50. Receipts: \$10 a day for the week of 6 days. Expenditures: Monday, \$7.50; Tuesday, \$12.25; Thursday, \$18.40; Saturday, \$21.43.

3. Cash on hand July 1, \$74.90. Receipts: July 6, \$60.70; July 15, \$50; July 21, \$18.90; July 31, \$50. Expenditures: July 10, \$25.50; July 7, \$6.50; July 16, \$40.59.

4. Cash on hand Oct. 1, \$185.50. Expenditures: Oct. 3, \$16.50; Oct. 11, \$89.70; Oct. 15, \$5.53; Oct. 25, \$17.75; Oct. 28, \$15.90; Oct. 30, \$12.

5. Jan. 1, '09, on hand \$75. Expenditures: Jan. 2, books \$3.50; Jan. 10, board \$7.50; Jan. 15, rent \$25; Jan. 25, clothes \$22.50. Jan. 30, salary received, \$250.

Balance the following accounts:

6.

7.

EXPENDI- TURES.	CASH.	EXPENDI- TURES.	CASH.
May 1, \$10.	On hand \$125.50	July 1, \$125.50	On hand \$1000
May 3 15.75		July 5 75.60	Profits 500
May 5 8.30		July 10 200.30	
May 10 12.60	Int. 20.00	July 16 190.05	Int. 300
May 15 1.25		July 25 400.25	Dividends 650
May 25 50.00	Salary 175.00	July 30 90.83	Profits 2010

REMITTANCES

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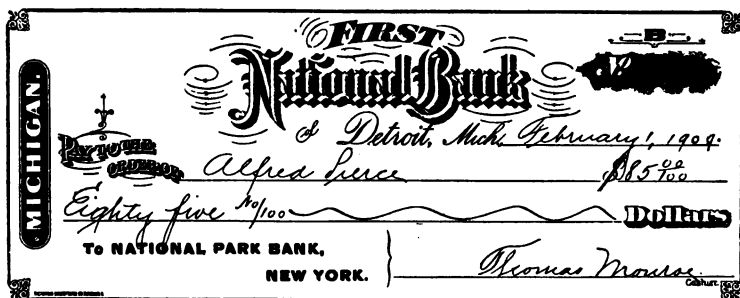
Checks

153. A check, drawn upon a bank in which the sender has a deposit, also may be sent. The person receiving the check may deposit it in a bank in his locality. That bank will collect the amount of the check from the bank on which it was drawn.

Many banks charge a small fee for collecting a check; consequently business houses object to this method of remittance, and it is customary to use some other form.

Bank Drafts

154. Money is frequently sent by mail by means of **bank drafts**. A bank draft is merely a check drawn by one bank upon another, and may have the form here shown.



The bank will charge Mr. Pierce \$85 and possibly a small fee in addition for this draft. Mr. Pierce will indorse this draft in full and mail it to Mr. Adams, to whom he wishes the money paid. The National Park Bank at New York will pay Mr. Adams \$85 on presentation of the draft. The indorsement is as here shown. If Mr. Adams desires, he may deposit this draft with some other bank for collection.

Pay to the order of

H. C. Adams.

Alfred Pierce.

4. Cash on hand Oct. 1, \$185.50. Expenditures: Oct. 3, \$16.50; Oct. 11, \$89.70; Oct. 15, \$5.53; Oct. 25, \$17.75; Oct. 28, \$15.90; Oct. 30, \$12.

5. Jan. 1, '09, on hand \$75. Expenditures: Jan. 2, books \$3.50; Jan. 10, board \$7.50; Jan. 15, rent \$25; Jan. 25, clothes \$22.50. Jan. 30, salary received, \$250.

Balance the following accounts :

6.

7.

EXPENDI- TURES.	CASH.
May 1, \$10.	On hand \$125.50
May 3 15.75	
May 5 8.30	
May 10 12.60	Int. 20.00
May 15 1.25	
May 25 50.00	Salary 175.00

EXPENDI- TURES.	CASH.
July 1, \$125.50	On hand \$1000
July 5 75.60	Profits 500
July 10 200.30	
July 16 190.05	Int. 300
July 25 400.25	Dividends 650
July 30 90.83	Profits 2010

REMITTANCES

Registered Mail

may be safely forwarded by mail in a letter at a post-office. The fee for this is 8 cents. delivery of the letter is returned to the

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ORAL EXERCISES

1. What is the premium at $\frac{1}{4}\%$ on a bank money order for \$1000? What is the total cost of such an order?
2. What is the total cost of an order for \$2,500 at $\frac{1}{8}\%$ premium?
3. Name two other kinds of money orders.
4. Name five ways of remitting money.

WRITTEN EXERCISES

1. Find the total cost of bank money orders for the following sums, premium $\frac{1}{4}\%$:

\$1,500	\$3,500	\$10,650	\$1,225
1,625	86.50	195.50	40.30

Commercial Drafts

The firm, Rider & Company of Duluth, owe D. Appleton & Company of New York \$350 for books. D. Appleton & Company may collect the account by means of the following draft:

<u>\$350.00</u>		<u>D Appleton & Company, No 1606</u>	
		<u>New York, February 1, 1909.</u>	
		<u>At sight pay to the order of</u>	
<u>Fourth National Bank</u>		<u>(New York)</u>	
<u>Three hundred fifty</u>		<u>1/100</u>	
<u>Dollars</u>			
<u>Value received and charge to</u>		<u>D. Appleton & Company,</u>	
<u>Rider & Co.</u>			
<u>Duluth, Minn.</u>		<u>John Smith</u>	
		<u>Treasurer</u>	

This is sent by the Fourth National Bank of New York to a bank of Duluth. This bank sends the draft by messenger to Rider & Co., who may accept the draft and pay the amount. If they refuse to pay it, the draft is returned to its maker.

157. Such a draft is called a **commercial draft**; and, since it was payable at sight, a **sight draft**. Instead of the words "at sight," the drawer may state any future time of payment. In that case, the draft is called a **time draft**. When the draft is presented, the debtor, if he intends to pay it, writes "Accepted" with date and signature across its face. An accepted time draft may be discounted like a note.

ORAL EXERCISES

1. John Doe draws a sight draft in favor of the Park National Bank of New York and charges the same to Richard Roe of Detroit, Michigan. In what bank does Mr. Roe deposit the draft for collection?

2. What kind of draft is the one mentioned in Exercise 1?

3. How would the Park National Bank proceed to collect this note?

4. What would become of the note if Richard Roe refused to accept it?

5. If it were a time draft for 60 da. and Mr. Roe accepted it, what would he do with the draft?

WRITTEN EXERCISES

1. The bank making the collection may charge a fee. What would be the cost of collecting a draft for \$589.75 at the rate of $\frac{1}{4}\%$?

2. A time draft for \$325 had been accepted; the holder discounted it 30 da. before it was due, at 6%. What were the proceeds?

3. What is the cost of collecting a draft for \$4,500 at $\frac{1}{4}\%$? What are the proceeds of the draft?

4. A time draft for \$1,500 had been accepted; the holder discounted it 2 mo. before it was due, at 6%. What were the proceeds?

INSURANCE

Fire Insurance

158. When a company insures a house for a certain sum—say \$2,000—against loss by fire, it agrees to make good the owner's loss to the extent of \$2,000, if the house is accidentally burned during the period specified.

159. In return, the owner pays the company in advance a stipulated amount called a **premium**.

160. The contract between the company and the owner is called a **policy**.

161. The amount for which the property is insured is called the **face** of the policy.

WRITTEN EXERCISES

1. A house is insured against fire for \$3,000; the rate is \$.65 per \$100 per annum. What is the annual premium? Two annual premiums in advance usually pay for 3 years' insurance. What is the amount paid in advance for three years' insurance on the house?

2. Mrs. Brown had her jewels and wardrobe insured against fire for \$3,000; the rate was \$.45 per \$100 per annum. What was the annual premium? Three annual premiums in advance usually pay for five years' insurance. What did Mrs. Brown pay for five years' insurance?

3. A cargo was insured for \$12,360 at $2\frac{1}{2}\%$. What was the annual premium?

4. A house valued at \$9,500 was insured for $\frac{3}{4}$ of its value at 3%. What was the annual premium?

5. The annual premium on a policy for insuring a block at 3% was \$465. What was the value of the block?

6. A stock of goods valued at \$2,500 was insured, and the annual premium was \$62.50. What was the rate of insurance?

7. A house was insured by a policy on which the premium was \$85.32 and the rate was 6%. What was the face of the policy?

Life Insurance

162. The essential feature of life insurance is, that the insurance company pays a fixed sum to the estate of a person named (**beneficiary**), whenever the insured person dies. In many cases the condition is added that the sum shall be paid to the insured himself, if living, at the end of a stipulated time; for example, 20 years. The first may be called a **life policy**; the second, a 20-year **endowment policy**.

163. In return, the insured person pays (usually annually in advance) a fixed sum called a **premium**.

The amount of the annual premium depends upon the age of the insured at the time when the payments are begun, and the character of the policy. It is usually stated at a rate per \$1000 of the insurance. There are many different kinds of policies, with different conditions and privileges. The following specimen table of rates may be used in the problems:

Age when policy is issued.	21	22	23	24	25
Annual premium, life.....	15.01	15.35	15.71	16.03	16.46
“ “ 20 yr...	42.44	42.50	42.55	42.61	42.66
Age when policy is issued.	26	27	28	29	30
Annual premium, life.....	16.87	17.31	17.76	18.24	18.74
“ “ 20 yr...	42.73	42.80	42.88	42.97	43.07
Age when policy is issued.	31	32	33	34	35
Annual premium, life.....	19.27	19.83	20.42	21.04	21.70
“ “ 20 yr...	43.17	43.29	43.41	43.67	43.73

WRITTEN EXERCISES

1. Mr. Gray at the age of 21 insured his life for \$1000 on the 20-year plan. At the expiration of the twenty years he was still living and received \$1000. How much more did he receive than he had paid in?

2. How much does a man pay, beginning at 35 years of age, for \$1000 of insurance on the life plan, if he lives to be 75 years of age?

3. Make and solve five problems about life insurance.

BROKERAGE

164. An agent who arranges for purchases or sales without actually receiving and delivering the goods is called a **broker**.

Thus, a broker in Chicago representing a wheat owner in Minneapolis may sell a carload of wheat in the Chicago market to another broker who represents a miller in Cincinnati. The wheat would be shipped from Minneapolis direct to Cincinnati after the transaction.

165. The fee paid the broker for buying or selling is called **brokerage**. It is usually a fixed sum per bushel, barrel, etc.

166. Some common rates of brokerage are:

For grain, $\frac{1}{8}$ ct. a bushel, the least charge being \$200.

For cotton, 5 cts. a bale, the least charge being \$500.

For pork, $2\frac{1}{2}$ cts. a barrel, the least charge being \$6.25.

For coffee, 4 cts. a bag of 130 lb.

ORAL EXERCISES

Find the brokerage at the above rates on each of the following sales:

1. 10,000 bu. wheat.

5. 2,000 bales cotton.

2. 100 bbl. pork.

6. 500 bags coffee.

3. 210,000 bu. wheat.

7. 4,000 bbl. pork.

4. 1000 bu. corn.

8. 5,000 bu. oats.

Solve:

9. 10 problems of your own, similar to those preceding.
10. A trader sold "10 wheat" (10,000 bu.) through a broker at 65 cts. a bushel. How much did he realize from the sale?
11. A firm bought 1000 bales of cotton on the Exchange at \$50 a bale. How much did it cost, including brokerage?
12. A broker bought for a customer 20,000 bu. of wheat at 65 cts. a bushel and sold it at 78 cts. How much did the customer gain, allowing the usual brokerage for buying and selling?

WRITTEN EXERCISES

1. Express the brokerage in oral exercises 10 and 11 above as a rate per cent.
2. In the market quotations, grain is listed per bushel, coffee is listed per pound, metal per ton, and cotton per bale of 500 lb. The following table contains the highest and lowest quotations of various staple commodities during a recent year:

	HIGHEST.	LOWEST.
Wheat.....	1.15 $\frac{1}{4}$.80
Corn.....	.81 $\frac{1}{2}$.52
Oats.....	.62 $\frac{1}{2}$.39
Coffee.....	.07 $\frac{1}{2}$.06 $\frac{1}{4}$
Iron.....	26.00	19.75
Tin.....	44.47 $\frac{1}{2}$	30.87 $\frac{1}{2}$
Copper.....	25.55	12.25
Lead.....	6.25	4.60
Cotton.....	13.55	10.55

Find the brokerage at $\frac{1}{4}$ ct. per bushel on 5,000 bu. of wheat and the total cost when bought at the highest price as quoted.

3. According to the table, how much was invested in wheat at the lowest price by a broker who received \$250?
4. Similarly, make and solve from this table five problems.

STOCKS AND BONDS

Stock Companies

167. When several persons wish to conduct a business enterprise together, they may organize a **company**.

168. The money invested in the business is called **capital**.

169. If the organization of the company is authorized by the state (in accordance with certain laws) the company is said to be **incorporated**. When incorporated, the company, **corporation**, can legally transact business like an individual.

170. The capital of a corporation is divided into equal amounts called **shares**. In this case the capital is called **capital stock**, or simply **stock**. Whatever amount a share represents is called the **face value**, or **par value**, of the share.

Capital is invested in a stock company by the purchase of one or more of the shares of stock.

171. The owners of stock are called **stockholders**, and certificates are issued for the shares of stock.

The illustration on the opposite page is a certificate of stock showing that Alfred Morgan owns 100 shares of the American District Telegraph Company.

172. The business is carried on under the management of officials elected by the stockholders. At regular intervals, usually twice or four times a year, the profits are divided among the stockholders, except what may be reserved for the needs of the business. The sum paid is called a **dividend**, and is usually expressed as a rate per cent of the face value of the share.

173. A stockholder may sell his stock to any purchaser for its face value, or for more or less. If it sells for its face value, it is said to be **at par**; otherwise, **above** or **below par**.



This Certificate that Alfred Morgan is the owner
of One hundred Shares of
One Hundred Dollars each, fully paid of the Capital Stock of

THE AMERICAN DISTRICT TELEGRAPH COMPANY OF MICHIGAN, LIMITED,
Transferable upon the books of the Company, and in accordance with the rules
and Regulations of said Company, on surrender of this Certificate.

[SEAL]

IN WITNESS WHEREOF, The Chairman and Secretary of said Company, by authority of the Board
of Managers, have hereunto subscribed their names, and caused the Seal of said Company to be
hereunto affixed, this fourth day of June 1899

Henry Schantz
Secretary

James Perkins
Cashier

When the price of stock is quoted 90, 85, 45, $37\frac{1}{2}$, or $284\frac{1}{2}$, it means that a share whose par value is \$100 can be bought for \$90, \$85, and so on.

There are "stock exchanges" in the large cities to facilitate the purchase and exchange of stocks. The stock broker charges a commission of $\frac{1}{2}$ of 1% on the *par value* for either buying or selling.

ORAL EXERCISES

In the exercises following, the par value of a share is to be regarded as \$100, unless otherwise specified.

1. What must one *add* to each quotation to find the cost of a share, including brokerage? What must one *subtract* to find what the sale of a share would net the seller?

2. Mr. Richmond owned 12 shares of New York Railway stock; he received an annual dividend of \$6 a share. What was his yearly income from these shares?

3. Mr. Marsh received a semiannual dividend of 4% on 10 shares of railroad stock. What was his annual income?

4. Mr. A owns 10 shares of stock on which a dividend of 3% has been declared. How much does he receive?

5. What is the annual income from 160 shares of stock that pays a quarterly dividend of $1\frac{3}{4}\%$?

WRITTEN EXERCISES

What is the cost of the following purchases of stock:

1. 17 shares of Baltimore and Ohio Railway stock at 105?

2. 150 shares of Illinois Central Railway stock at 128?

3. 2,000 shares of St. Louis and S. W. stock at 128?

4. 1,500 shares of Niagara Power Co. stock at $84\frac{1}{2}$?

5. 5,000 shares of Texas Pacific Railway stock at $41\frac{1}{2}$?

6. What did the owner of the stock in each of the above exercises realize from its sale, after having paid the broker?

174. To distinguish the rate of dividend from the rate of earnings per dollar invested, the latter is called the *rate of income on the investment*.

EXAMPLE: A man bought 1 share of railroad stock at \$90, including brokerage; the stock paid an annual dividend of 4%. What was the man's rate of income on his investment?

The man received \$4 per \$90 invested, or $4\frac{1}{3}$ cts. per dollar, or $4\frac{1}{3}\%$ income on the investment.

WRITTEN EXERCISES

1. If Diamond Match stock sells for 140 and pays an annual dividend of 10%, find the rate of income on the investment.

2. What is the rate of income on an investment in 10 shares of stock paying 4% annually and bought at 80?

3. A company whose stock is listed at $49\frac{1}{2}$ declares an annual dividend of \$4.75 a share. What is the rate of income?

175. Sometimes the capital stock of a corporation is divided into two kinds, called **preferred stock** and **common stock**. A dividend must be paid on the preferred stock, before any dividend whatever may be paid on the common stock.

ORAL EXERCISES

1. A carpet company organized with a capital of \$1,000,000. How many 100-dollar shares of stock were there?

2. The organizers subscribed for $\frac{1}{2}$ of the stock. How many shares were sold to other investors?

3. 2,000 shares were preferred stock. How many shares of common stock were there?

4. What sum was required annually to pay the dividends on the preferred stock at 5%?

5. At the end of the first year a 2% dividend was declared on the common stock. How much was this a share? What sum was paid as dividends on the common stock?

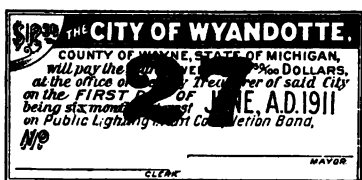
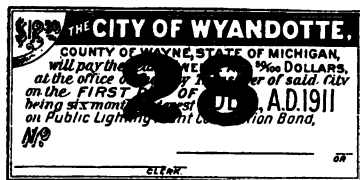
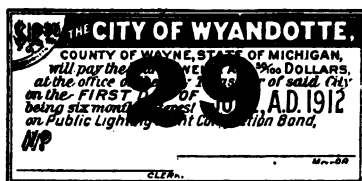
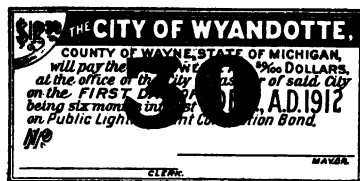
Bonds

176. When the Government (national, state, county, or city) or a corporation borrows large sums of money for a long time, it acknowledges the receipt of the money and promises to repay it in a statement called a **bond**. A bond is usually issued for each \$1000 borrowed. The opposite page shows a bond whose price is \$500.

177. The amount of the bond is called its **par value**.

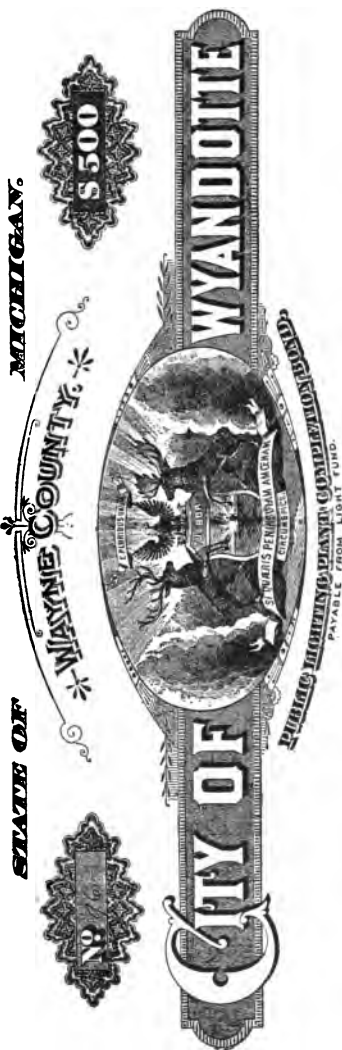
Thus, the face value of the above bond is \$500.

178. Bonds bear interest at a specified rate. This is usually paid upon presentation of certificates called **coupons** attached to the bonds. The coupons are promissory notes, which, if not paid when due, bear interest at the legal rate.



ORAL EXERCISES

1. What is the face of the bond shown on page 157? What is the annual interest at 5%?
2. The coupons above are the last 4 of the 30 coupons of this bond. Each represents the interest for 6 months. What was the whole amount of interest on the bond?



Know All Men by These Presents, That the CITY OF WYANDOTTE, in the County of Wayne, State of Michigan, acknowledge itself justly indebted and promises to pay to the Bearer,

lawful money of the United States of **FIVE HUNDRED DOLLARS**, upon the presentation of this bond, on the 15th Day of June and the First Day of December of each and every year until the principal and interest shall be paid in full.

THIS BOND is issued in accordance with the Land in the State of Michigan, which entered the City of Wyandotte, and is authorized by the Council of the City of Wyandotte, and the Common Council of said City, and is subject to the payment of said City, and the particular public improvement to raise money for which this bond is issued, in the completion of the erection and equipment of the Public Light Plant of said City.

IN TESTIMONY WHEREOF, The Mayor and the Clerk of said City of Wyandotte have signed and countersigned this Bond and affixed the seal of said city this First Day of December, A. D. 1897.

James M. Carmichael
Richard B. Hart

179. Bonds differ from stock in that the bonds are secured by the property of the company and bear a permanent rate of interest.

The interest must be paid on the bonds before any dividend can be paid on the stock. A bondholder is a creditor; a stockholder is a part owner. Bonds, like stocks, are bought and sold in the market.

When bonds are purchased at prices above or below par and held until maturity, the rate of income, or yield, differs from the rate of interest. But this phase of the subject is too difficult to present here.

180. The broker's commission for buying or selling bonds is $\frac{1}{8}$ of 1% of the *par value*.

Amounts bought and sold are stated in par value, and the price is quoted at a rate per \$100 of par value. The rate of interest is usually added.

Thus, U. S. 4's 142 means that United States bonds bearing 4% interest on par value are sold at \$142 for each \$100 of par value.

EXAMPLE: What is the cost of \$3,000 (par value) Erie Canal bonds at 101?

- Solution:* (1) Each \$100 of par value costs \$101.
 (2) \$3,000 of par value cost $30 \times \$101$, or \$3,030.
 (3) Brokerage, $\frac{1}{8}$ of 1% of par value, \$3.75.
 (4) Total cost = \$3,033.75.

WRITTEN EXERCISES

- What is the cost of \$15,000 city bonds at 105?
- Find the proceeds from each of the following sales:

\$5,000 Central Union Gas Co. of N. Y. 1st mortgage 5% bonds. 108 $\frac{1}{8}$ \$10,000 Union Gas Light Co. of Kings Co. 1st mortgage 6% bonds..... 106 $\frac{3}{8}$	\$20,000 Northern Pacific Gt. Northern R. R. (C., B. & Q. coll.) 4% bonds..... 94 $\frac{1}{8}$ \$6,000 Southern R. R. 1st mort- gage 5% bonds..... 115 $\frac{3}{8}$
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- Find the amount of annual interest from each purchase above.
- Find the brokerage on each of the above sales.

REVIEW

ORAL EXERCISES

1. Mr. Carelton of Detroit owes Rice and Co. of St. Louis \$175 for some goods. How may he discharge the indebtedness? Will he be likely to use an express or a postal order or a bank draft?

2. What is a bank draft? What is a commercial draft?

3. What is a sight draft? A time draft? How is a commercial draft collected?

4. Smith & Co. of New York draw upon Jones & Co. of Johnstown for \$250, 30 days after sight. What may Jones & Co. write on the face of the draft to show that they will pay it?

5. What is the usual rate of brokerage for buying and selling grain on the Exchanges? At this rate find the brokerage on the following purchases:

32,000 bu. of wheat;	10,000 bu. of barley;
20,000 bu. of oats;	7,000 bu. of corn.

6. A firm bought 5,000 bales of cotton at \$50 a bale, brokerage 5 cts. a bale. What did it cost?

7. What is meant by the stock of a corporation? What is the usual par value of stock? Name a price that is above par.

8. What is a dividend? On what value of the stock is it declared?

9. Where are stocks usually bought and sold, and through whom? What is the fee for buying or selling called?

10. What is meant by rate of income on the investment? How is it computed?

11. What is a bond? What is usually attached to a bond to represent the interest due?

12. Where are bonds usually bought and sold? What must one add to the quoted price of a bond to find its cost?

WRITTEN EXERCISES

1. Write an order to Sargent and Smith, 18 Grand St., New York, for a tool chest which they advertise in "The Youth's Companion" for \$8.95, purchaser to pay carriage. State in what form you send the money and give directions as to sending the tool chest to you. (When ready for shipment it weighs 65 lb. If convenient, ascertain which would be the cheaper way of having it come to you, by freight or by express, remembering that you must also pay cartage from the freight depot to your home, if sent by freight.)

2. Make and receipt a bill for 5 tons of hard coal at \$6.75, plus 25 cts. per ton for carrying it to the bin; 1 load of kindling wood, \$1.25.

3. Mr. Johnson bought 100 shares of Pennsylvania Railroad stock at 130. How much did he pay his broker? How much did the seller receive from his broker?

4. If New York Central stock pays a dividend of 5%, what is the income from 238 shares?

5. What is the income from 250 shares of stock paying a dividend of $4\frac{1}{2}\%$?

6. A man purchased some stock at \$110 a share, including brokerage, which paid a dividend of $5\frac{1}{2}\%$. What was the rate of income on the investment?

7. A premium of \$157.50 was paid on a policy whose face was \$4,500. What was the rate of insurance?

8. What is the cost of 500 Metropolitan bonds (par \$100) quoted at 109 $\frac{3}{4}$?

9. A man bought 10 bonds (par \$100) quoted at 149 $\frac{3}{4}$ and paying an annual interest at the rate of $4\frac{1}{2}\%$. What was the amount of his annual interest?

CHAPTER VIII

POWERS AND ROOTS

POWERS

181. The product of two equal factors is called the **second power**, or **square**, of the repeated factor.

Thus, 4, the product of 2 and 2, is the second power, or square, of 2.

182. The product of three equal factors is called the **third power**, or **cube**, of the repeated factor.

Thus, 8, the product of 2 and 2 and 2, is the third power, or cube, of 2.

183. The product of four equal factors is called the **fourth power** of the repeated factor, and so on.

Thus, 16, the product of 2, 2, 2, and 2, is the fourth power of 2.

Similarly, 32 is the fifth power of 2, 64 is the sixth power, and so on.

ORAL EXERCISES

1. 25 is what power of 5?
2. 125 is what power of 5?
3. 27 is what power of 3?
4. 81 is what power of 3?
5. What is the area of a square 2 in. on a side?
6. What is the area of a square 5 yd. on a side?
7. What is the volume of a cube 3 ft. on an edge?

184. Powers of numbers can be denoted by exponents.

Thus, The second power of 2 (or 4) = 2^2 .

 The third power of 2 (or 8) = 2^3 .

 The fourth power of 3 (or 81) = 3^4 .

185. *A power of any number is found by multiplication.*

WRITTEN EXERCISES

Find the following squares:

- | | | | |
|-------------------------|-------------|------------------------|--------------|
| 1. 15^2 . | 3. 11^2 . | 5. 8^2 . | 7. 2.5^2 . |
| 2. $(\frac{9}{16})^2$. | 4. 25^2 . | 6. $(\frac{3}{4})^2$. | 8. $.1^2$. |

Find the volume of a cube whose edge is:

- | | | | |
|-----------|-------------|------------|-----------|
| 9. 4 in. | 11. 1.2 in. | 13. .6 in. | 15. 4 yd. |
| 10. 5 ft. | 12. 3 ft. | 14. 9 ft. | 16. 7 ft. |

SQUARE ROOTS

186. Any number is the square root of its square.

Thus, 4 is the square root of 4^2 , or 16.
 1.5 is the square root of 1.5^2 , or 2.25.
 $\frac{3}{4}$ is the square root of $(\frac{3}{4})^2$, or $\frac{9}{16}$.

187. A root of a number may be indicated by placing over the number the symbol $\sqrt{\quad}$, called the **radical sign**.

Thus, the square root of 25 is denoted by $\sqrt{25}$. The cube root of 8 by $\sqrt[3]{8}$; the fourth root of 16 by $\sqrt[4]{16}$, and so on.

To Find the Square Root by Factoring

188. When it is possible to find the square root by factoring, factor the number into its prime factors and arrange them in two equal sets. Either set is the square root of the number.

EXAMPLE: Find $\sqrt{225}$.

$225 = 3 \times 5 \times 3 \times 5 = 15 \times 15 = 15^2$. Therefore, $\sqrt{225} = 15$.

WRITTEN EXERCISES

Find by factoring the square roots of:

- | | | | | |
|---------|-----------|---------|------------|----------|
| 1. 144. | 4. 961. | 7. 625. | 10. 2,601. | 13. 576. |
| 2. 400. | 5. 289. | 8. 441. | 11. 324. | 14. 169. |
| 3. 256. | 6. 9,801. | 9. 196. | 12. 3,600. | 15. 900. |

To Find the Square Root when the Number Cannot Easily Be Factored

189. A knowledge of the way in which a square is formed will assist in finding square roots.

EXAMPLE: What is the square of 14?

$$14^2 = 14 \times 14 = 196.$$

But $14 = 10 + 4$, and we may multiply $10 + 4$ by $10 + 4$, as follows:

$$\begin{array}{r} (10 + 4) \times 10 = 10^2 + 10 \times 4 \\ (10 + 4) \times 4 = \quad \quad 10 \times 4 + 4^2 \\ \hline 10^2 + 2 \text{ times } 10 \times 4 + 4^2 \end{array}$$

$$\text{Then, } (10 + 4) \times (10 + 4) = 100 + 80 + 16 = 196.$$

The work above shows that the square of a number composed of tens and units is the square of the units plus twice the product of the tens and the units plus the square of the tens.

$$\begin{aligned} \text{Thus, } 15^2 &= 10^2 + 2(10 \times 5) + 5^2 \\ &= 100 + 100 + 25 = 225. \end{aligned}$$

190. We can determine in advance the number of figures in the square root of the square of an integer.

$$\begin{aligned} 1. \text{ Read: } 1^2 &= 1. & 5^2 &= 25. & 9^2 &= 81. \\ & & 2^2 &= 4. & 7^2 &= 49. & 10^2 &= 100. \end{aligned}$$

A one-figure integer has at most how many figures in its square? What is the least number it can have?

$$\begin{aligned} 2. \text{ Read: } 10^2 &= 100. & 25^2 &= 625. & 99^2 &= 9,801. \\ & & 15^2 &= 225. & 50^2 &= 2,500. & 100^2 &= 10,000. \end{aligned}$$

A two-figure integer has at most how many figures in its square? What is the least number it can have?

3. Similarly, the square of an integer of 3 figures is composed of 5 or 6 figures, and so on.

Thus, separate the square into periods of 2 figures each, beginning at the decimal point, and the number of places is the number of figures in the root. The place at the left may not be complete. For example, 1'69'00 has 3 periods; hence the square root contains 3 places.

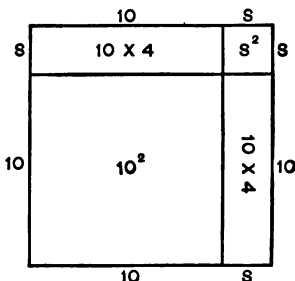
191. EXAMPLE: Find the square root of 196.

Our knowledge of how squares are formed will help to find the square root. Consider 196 to be the area of a square of which we wish to find the side.

By marking off the periods in 1 96 we see that the number contains the square of 10.

Subtracting this, there is left 96, which shows that the side of the square sought is more than 10. If we call S the part to be added to the side already found, we must make the additions shown in the figure in order to form a new square.

Thus, 96 contains $2 \times (10 \times S) + S^2$. We shall find S approximately by dividing 96 by 2×10 . The result is 4. Using this for S , $2 \times 10 \times S + S^2$ becomes $2 \times 10 \times 4 + 16 = 96$.



Subtracting 96, the remainder is zero; hence we have found the side, 14, of a square whose area is 196. That is, we have found the square root of 196 to be 14.

192. To find the square root of any number:

1. *Point off the number into periods of two figures each, beginning at the right (at the decimal point in a number containing a decimal).*

2. *By inspection find the largest integer whose square is not greater than the left period.*

(In Example A it is 9.)

3. *Use this integer as the first digit of the root. Subtract its square from the left period.*

(In Example A this square is 81.)

4. *Bring down the next period.*

(In Example A this makes 364.)

	(A)
Root	9 2
Number	84'64
	81
	<hr/>
18	364
182	<hr/>
	364

5. Multiply the part of the root already found by 2. This number is called the **trial divisor**. (18 in Example A.)

6. Divide the remainder (omitting the right digit) by the trial divisor and use the digit found as the next digit of the root.

(In Example A, $36 \div 18 = 2$.)

7. Annex this digit to the trial divisor. This forms the **complete divisor**. (182 in A.)

8. Multiply the complete divisor by the digit of the root just found and subtract.

NOTE.—It may happen that the product to be subtracted is larger than the number from which it is to be subtracted. This indicates that the trial divisor gave too large a digit. Try the next smaller digit.

9. Repeat steps 4 to 8 until all the periods have been brought down.

	(B)
Root	3 0. 6 9
Number	9'41'87'61
	9
6	41
60	4187
606	3636
612	55161
6129	55161

	(C)
Root	1 4. 1 4 +
Number	2'00
	1
2	100
24	96
28	400
281	281
282	11900
2824	11296
	604

If a zero occurs in the root, as in Example B, when 4 is not divisible by 6, bring down the next period and proceed as before.

If the last remainder is zero, as in Example B, the process is ended, the given number is a perfect square, and its root has been found exactly. If the last remainder is not zero, as in (C), the process may be continued as far as desired by supplying zeros.

Test: The square of the root should equal the given number.

193. To extract the square root of a fraction, either reduce the fraction to a decimal and proceed, or extract the square root of both terms.

WRITTEN EXERCISES

Extract the square root of:

- | | | | | |
|--------------------|----------------------|----------------------|------------------------|-------------|
| 1. 729. | 8. 841. | 15. 22,500. | 22. $\frac{49}{100}$. | 29. 156.25. |
| 2. 72.25. | 9. $\frac{1}{4}$. | 16. 784. | 23. 1,444. | 30. 3,136. |
| 3. 1.69. | 10. 5,625. | 17. 20.25. | 24. 2.56. | 31. .2025. |
| 4. $\frac{1}{4}$. | 11. 4.41. | 18. $\frac{1}{16}$. | 25. 1,681. | 32. 8,836. |
| 5. 1,225. | 12. 625. | 19. 7,225. | 26. 3,025. | 33. 96.04. |
| 6. 6,889. | 13. $\frac{1}{16}$. | 20. 2.25. | 27. $\frac{1}{16}$. | 34. 1,936. |
| 7. 361. | 14. 4,225. | 21. 2,025. | 28. 11,025. | 35. 6.25. |

Solve:

36. How long is each side of a square court of 1,225 sq. yd.?
37. What is the perimeter of a field 16 rd. wide and 144 rd. long? What would be the perimeter if the field were square and had the same area?
38. What is the distance in rods around a square tract of land containing 1,681 sq. rd.?
39. A square field of corn contains 252,004 hills of corn. How many rows in the field?
40. How long is each side of a square floor of 7,225 sq. ft.?

Find the following roots to 3 places of decimals:

- | | | | |
|-------------------|--------------------|--------------------|---------------------|
| 41. $\sqrt{2}$. | 45. $\sqrt{3}$. | 49. $\sqrt{5}$. | 53. $\sqrt{7}$. |
| 42. $\sqrt{11}$. | 46. $\sqrt{2.5}$. | 50. $\sqrt{.02}$. | 54. $\sqrt{1.05}$. |
| 43. $\sqrt{13}$. | 47. $\sqrt{.3}$. | 51. $\sqrt{.05}$. | 55. $\sqrt{75}$. |
| 44. $\sqrt{15}$. | 48. $\sqrt{.2}$. | 52. $\sqrt{1.5}$. | 56. $\sqrt{.75}$. |

Solve:

57. What is the length in rods of a side of a square field containing 1 acre? Find the result to two decimal places.

CHAPTER IX

MENSURATION

THE RIGHT TRIANGLE

194. A triangle one of whose angles is a right angle is called a **right triangle**.

The side opposite the right angle of a right triangle is called the **hypotenuse**.

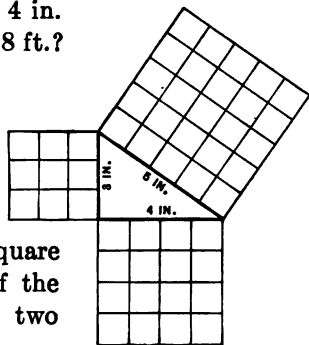
ORAL EXERCISES

1. What is the area of a square 4 in. on a side? 5 in.? 3 in.? 9 in.? 8 ft.? 10 yd.?

2. What is the length of each side of the triangle in the figure?

3. What is the area of the square drawn on the longest side?

4. Compare the area of the square on the longest side with the sum of the areas of the squares on the other two sides.



195. In any right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

If h represents the length of the hypotenuse, and a and b those of the sides, then $h^2 = a^2 + b^2$.

This relation is called the *Pythagorean Theorem*, after the Greek philosopher and mathematician, Pythagoras (about 500 B.C.), who is supposed to have discovered it.

WRITTEN EXERCISES

Compute the hypotenuse of a right triangle having the following sides forming the right angle:

1. 9 in., 12 in. 2. 5 in., 12 in. 3. 12 in., 16 in.

4. Compare the square on the hypotenuse with the sum of the squares on the other two sides.

196. PREPARATORY.

The hypotenuse of a right triangle is 10 in. Find the area of the square having this line for a side. The length of another side is 6 in. What is the area of the square on this side? What is the area of the square on the third side of the triangle? How long is the third side of the triangle?

EXAMPLE: Find the length of the hypotenuse of a right triangle whose other two sides are 8 ft. and 15 ft. respectively.

$$\text{Solution: } h = \sqrt{8^2 + 15^2} = \sqrt{289} = 17.$$

The hypotenuse is 17 ft.

WRITTEN EXERCISES

Find the unknown sides of the following right triangles:

	1.	2.	3.	4.	5.
Hypotenuse:	5	x	13	10	x
Side:	4	3	x	6	6
Side:	x	4	12	x	8

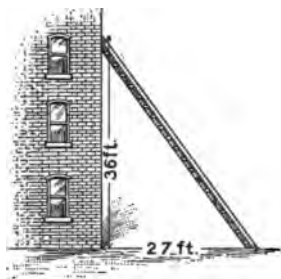
Solve:

6. The width of the end of a house was 24 ft., and the height of the ridge of the roof above the level of the eaves was 12 ft. Find the length of the slope of the roof from the ridge to the eaves.

7. A garden has the form of a right triangle. Its longest side is 149 ft., and one side is 51 ft. What will it cost to surround it with a fence at \$.70 a running foot?

8. A garden in the form of a right triangle has a hypotenuse 50 ft. long, and one side 30 ft. long. At \$.70 a running foot, what is the cost of a fence enclosing the garden?

9. A rope 73 ft. long is fastened to the top of a boat's mast, and when drawn taut, it touches a point on the deck 55 ft. from the foot of the mast. Find the height of the mast.



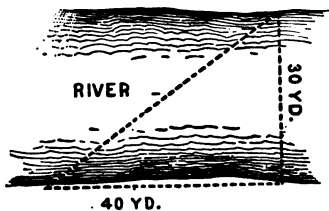
10. A ladder is placed against a building, as shown in the figure. How long is the ladder?

11. If a ladder 41 ft. long rests against a wall with its foot 9 ft. from the wall, how high is the top of the ladder from the ground?

12. A mason measures from one corner along the adjacent sides of a wall 30 ft. and 40 ft. He then connects the ends of these lines to see if the triangle thus formed is a right triangle. If it is, what is the length of this third line?

13. A rectangular tract of land was 3 mi. on one side and 4 mi. on the adjacent side. What was the distance diagonally from one corner of the tract to the other?

14. A swimmer in crossing a stream 30 yd. wide landed 40 yd. farther down stream. How many yards was it from his starting point to the landing place?



15. A trunk is 3 ft. long and 2 ft. wide. What is the longest umbrella that can be laid flat on the bottom of the trunk?

CIRCLES

197. PREPARATORY.

The distance through a round body, as a pencil or a drinking cup, is called its **diameter**, and may be measured by use of triangles and a scale as shown in Figure 1.



Fig. 1.

1. Measure the diameters of several round objects and record the results.

2. Wrap a piece of paper about each object until the ends overlap, as in Figure 2. Prick the ends

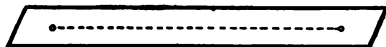


Fig. 3.

Figure 3. This is the distance around the object and is called its **circumference**.

with a pin. Release the paper and measure between the pinholes, as in

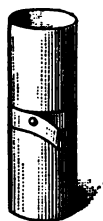


Fig. 2.

3. Find the ratio of the circumference of each to its diameter.

198. A **circle** is a closed curve every point of which is at the same distance from a fixed point called the **center**. The straight line joining the center and any point of the circle is called a **radius**.

199. The ratio of the circumference of a circle to its diameter is represented by the Greek letter π (pronounced pi). The value of π can be found approximately only; 3.14 is correct to the nearest hundredth, and should be used in the following problems.

200. If d denotes the diameter of a circle, πd denotes its circumference. If r denotes the radius, $d = 2r$, and $\pi d = 2\pi r$, or circumference $= 2\pi r$.

For example, the circumference of a circle 8 in. in diameter is approximately 3.14×8 in., or 25.12 in.

The circle is often defined as a surface, but the definition on page 170 is preferable because it is the one used in geography, advanced mathematics, and common parlance. The area enclosed by a circle is called the area of the circle.

WRITTEN EXERCISES

1. Find the circumference of a circle of diameter 6 in. Of diameter 1 ft. Of radius 6 in. Of radius 3 yd.
2. If the circumference of a circle is given, by what must it be divided to produce the diameter?
3. Find the diameter of a circle of circumference 3.14 ft.
4. Find the length of the radius of a circle whose circumference is 6.28 in. Of one whose circumference is 9.42 yd.
5. How many times will a bicycle wheel 28 in. in diameter revolve in going 1 mi.?

201. PREPARATORY.

Cut out of paper a circle of radius 3 in. and cut along the lines marked in Figure

1. Open out the circle as shown in Figure 2. The parts are called *sectors*.



Fig. 1.



Fig. 2.

202. A **sector** of a circle resembles a triangle, the curve being the base, and a radius the altitude.

203. It is shown in geometry that the area of a sector is found by taking one-half of the base times the altitude.

204. Denoting the base of a sector by b and its altitude by r , the area is $\frac{br}{2}$.

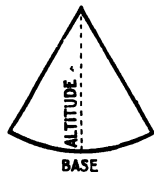


Fig. 3.

As all the sectors of a circle have the same altitude, the area of all of them (and, therefore, the area of the circle) is evidently the sum of their bases times half of the common altitude. The

sum of all the bases is the circumference, since the circumference was cut up to make them. Therefore, *the area of a circle = circumference $\times \frac{1}{2}$ radius = $2\pi r \times \frac{r}{2} = \pi r^2$.*

205. *The area of a circle is π times the square of the radius.*

WRITTEN EXERCISES

1. Find the length of a circle of radius 4 in., using 3.14 for π . Find its area.
2. Find the area of circles of radii: 6 in.; 1 ft.; 2 in.; 2 ft.; 10 ft.; 10 in.
3. The area of a circular garden was 12.56 sq. rd. What was the diameter of the garden?
4. What was the cost of a fence around it at 30 cts. per foot?
5. The area enclosed by a circular walk is 28.26 sq. yd. What is the radius of the circular area?
6. Find the area included within a semicircle and a diameter, the diameter being 5 ft.

7. The rectangular court shown in the picture is broken by a circle in the center, by semicircles at the ends, and by a trapezoid at the side. From the dimensions given in feet, find the area of the court not occupied by these spaces.

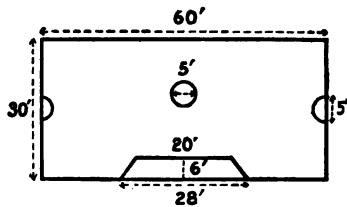


Fig. 1.

8. If a figure is drawn as indicated by Figure 2, what is the area of the square?
9. What part of a circle occupies each corner? What is the area of these parts?

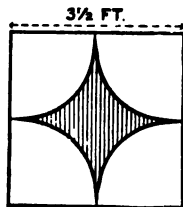


Fig. 2.

10. Draw a figure like that shown for Exercise 8, using a square of side 6 in. What is the area of the shaded part?

11. If the circular tiles shown in Figure 1 are 1 in. in diameter, what is the area of that part of the rectangle not covered by the tiles?

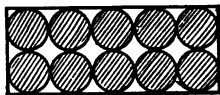


Fig. 1.

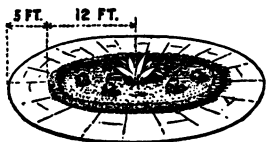


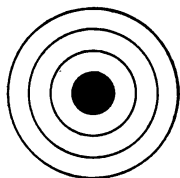
Fig. 2.

12. A walk of uniform width surrounds a circular grass plat, as shown in Figure 2. Find the area of the plat. Of the plat and the walk together. Of the walk.

13. What is the cost of a floor for a round tower 40 ft. in diameter at 32 cts. per square foot?

14. A round tree trunk has a circumference of 14.13 ft. What is its diameter? What is the area of a cross section?

15. A target is 8 ft. in diameter, the bull's-eye is 2 ft. in diameter, and each of the rings formed by the other circles is 1 ft. wide. What part of the whole target is the bull's-eye? What part of the whole is each ring?



16. The rotunda of the capitol in Washington is 98 ft. in diameter. Find the cost of a floor for it at 49 cts. per square foot.

17. A semicircular platform 20 ft. in diameter is to be carpeted with carpet 1 yd. wide, costing \$1.20 a yard. Allowing 5% for waste in fitting, what is the cost?

18. Find the cost of clearing a circular skating space 500 ft. in diameter, if the snow is 7 in. deep on the ice, and the cost of removal is 5 cts. a cubic yard.

19. A cow is tethered in the center of a field 100 ft. square with a rope 50 ft. long. Over how many square feet can she graze? Over what fractional part of the field?

20. The earth is about 8,000 mi. in diameter and makes one rotation on its axis in 24 hr. How far does a point on the equator move in a day? In an hour? In a minute?

HEXAGONS

206. The **regular hexagon** occurs frequently in nature and is much used in architecture and designs.

It is constructed by drawing a circle and drawing successive chords equal to the radius. (See Figure 1.)

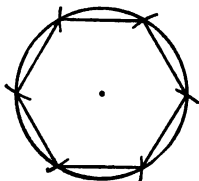


Fig. 1.

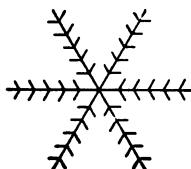


Fig. 2.

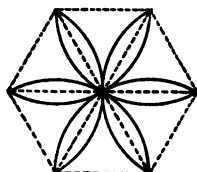


Fig. 3.

WRITTEN EXERCISES

1. Construct a regular hexagon, using dotted lines for the sides.

2. From the center of the figure draw full lines to the corners. Along each such line draw short parallel lines as shown in Figure 2. The resulting figure is a common pattern among snow crystals.

3. Draw a regular hexagon in dotted lines. By using each vertex as a center, draw the curves shown in Figure 3. Complete the figure and erase the dotted lines.

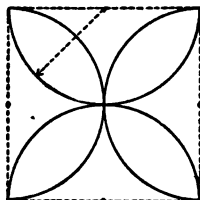


Fig. 4.

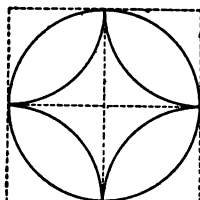
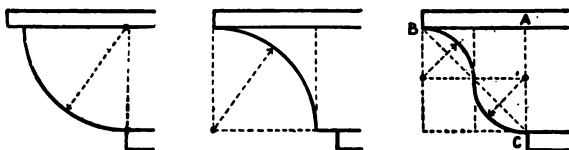


Fig. 5.

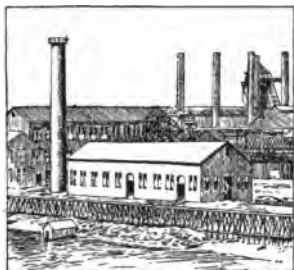
4. Show how to draw the designs of Figures 4 and 5.



5. Explain from the figures how to construct a design for each of these moldings.

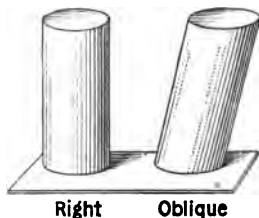
CYLINDERS

207. Curved surfaces, like those of sections of stovepipe, factory smokestacks, and funnels of ships, are called **cylindrical surfaces**.



208. If the ends of a cylindrical surface are in parallel planes, the whole is called a **cylinder**. The ends are called the **bases**. If the base is a circle, the cylinder is called **circular**.

209. Right and Oblique Cylinders. If the curved surface is perpendicular to the plane of the base, the cylinder is called a **right cylinder**. Cylinders that are not right cylinders are **oblique cylinders**.



We shall treat only right circular cylinders, and shall call them merely cylinders.

210. PREPARATORY.

1. Wrap a piece of paper around a cylinder. Cut it so that the edges meet. Trim the ends even with the cylinder. Unroll the paper. What kind of figure is the paper?

2. Measure the length and breadth of the paper and find the area. What is the area of the curved surface of the cylinder? How does the length of the paper compare with the distance around the cylinder?

211. The surface of an object exclusive of its ends is usually called its **lateral surface**.

212. *The lateral surface of a right cylinder is the product of its altitude and the circumference of its base.*

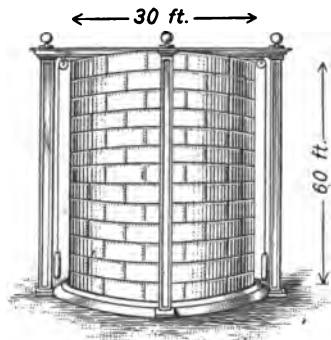
WRITTEN EXERCISES

Find the numbers to fill the blanks :

DIAMETER.	ALTITUDE.	LATERAL AREA.
1. 4 in.	10 in.	()
2. 2 in.	()	31.4 sq. in.
3. ()	5 in.	62.8 sq. in.

Solve :

4. Find the area of the curved surface of the cylindrical gas tank shown in the picture.



5. What will it cost to paint the surface mentioned in Exercise 4 at \$.50 a square yard? Test.

6. How many square feet are there in the top and curved surface of a gas tank 36 ft. in diameter and 45 ft. high?

7. What will it cost to paint the top and curved surface of the tank of Exercise 6 at \$.45 a square yard?

213. *The volume of a cylinder is the product of its altitude and the area of its base.*

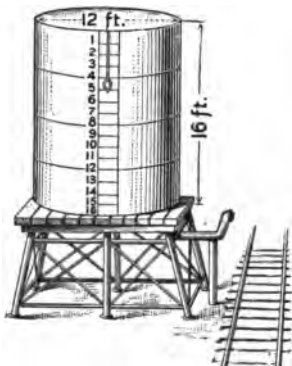
WRITTEN EXERCISES

1. How many cubic feet in a cylindrical tank 1 ft. high, whose base covers 40 sq. ft.? In a tank 2 ft. high having the same base? In one 5 ft. high?

2. Find the area of the base of the water tank shown in the picture. From the height given, find the volume in cubic feet.

3. Taking 1 cu. ft. as $7\frac{1}{2}$ gal. how many gallons does this tank hold?

4. When the gauge reads 5, it means that the water is 5 ft. deep in the tank. How many gallons are there in it then?



Find the numbers to fill the blanks for these cylinders :

DIAMETER.	ALTITUDE.	VOLUME.
5. 3 in.	8 in.	()
6. ()	5 in.	62.8 cu. in.
7. 1 ft.	()	12.56 cu. ft.

8. Find the number of square inches of iron in a stovepipe 3 ft. long and 6 in. in diameter.

9. What will it cost to paint the surface of a standpipe 52 ft. in circumference and 68 ft. high, with open top, at 35 cts. per square yard?

10. How many cubic yards of water will the pipe hold?

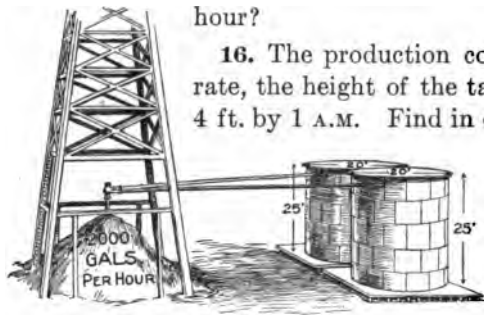
11. The pit for a turntable is 24 ft. in diameter and 4 ft. deep. Find the cost of excavation at 70 cts. per cubic yard.

12. A sprinkling wagon has a cylindrical tank 12 ft. long and 4 ft. in diameter. How many gallons does it contain? (Use $7\frac{1}{2}$ gal. = 1 cu. ft.)

13. How many gallons will the standpipe in Exercise 9 hold?

14. A drain pipe has an internal diameter of 30 in. It is half full of water which flows at the rate of 2 mi. per hour. How many cubic feet of water pass any point per minute?

15. At 7 A.M. a gas tank 220 ft. in circumference is 10 ft. high. At 5 P.M. it has been raised to the height of 40 ft. by the manufactured gas, none having been consumed. How many cubic feet of gas have been manufactured per hour?

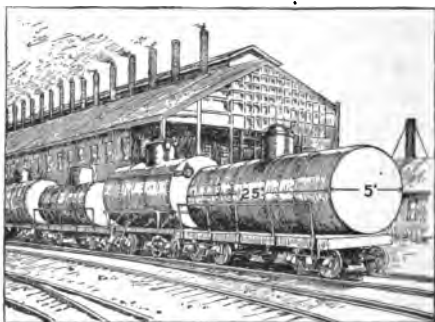


16. The production continuing at the same rate, the height of the tank had diminished to 4 ft. by 1 A.M. Find in cubic feet the average hourly consumption from 5 P.M. to 1 A.M.

17. How many gallons of petroleum does a well flowing at the rate mentioned in the picture produce in a day? In a week? In a year?

18. What is the capacity of the tanks?

19. A pipe-line for carrying oil from Pittsburgh to New York City delivers 300 gal. a minute. How many gallons does it deliver in a day? In 365 days?



How long would it take to empty 100 tanks like those above?

20. The refined oil is shipped in tank cars. How many gallons does this car carry? What is the value of such a carload of kerosene at 9 cts. a gallon? Of gasoline at 13 cts. a gallon?

21. A wagon for watering streets had a cylindrical tank 12 ft. long and $3\frac{1}{2}$ ft. in diameter. How many gallons did it hold?

PYRAMIDS

214. If pieces of paper be cut like Figures 1 and 2 and folded so that the edges of the triangles come together, the forms made are **pyramids**.

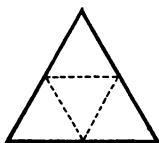


Fig. 1.

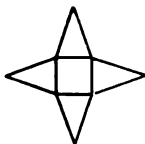


Fig. 2.



Fig. 3.



Fig. 4.

215. The base, or bottom, of a pyramid may have any number of sides. Pyramids are called *triangular*, *square*, or the like, according to the shape of the base. What kind of figures are the other faces of a pyramid?

216. PREPARATORY.

1. A pupil wrapped a piece of paper around a square pyramid and trimmed it off even with the base. He also wrapped a piece of paper around a rectangular block with base and altitude equal to those of the pyramid, leaving one end open and trimmed off even with the solid. After pasting the margins so as to hold the forms firmly in shape, he filled the paper pyramid with sand and then poured the sand into the paper prism. By doing this three times, he filled the prism.

2. If convenient, perform an experiment similar to this.

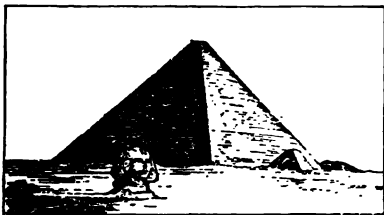
3. The volume of the pyramid was found to be three times as great as that of the prism. Their bases and altitudes were equal. Hence, the volume of a square pyramid is $\frac{1}{3}$ of its base and altitude.

217. *The volume of any pyramid, no matter what its base, is $\frac{1}{3}$ of the product of the altitude and the area of the base.*

WRITTEN EXERCISES

1. Find the volume of a triangular pyramid, the area of whose base is 36 sq. in. and whose altitude is 8 in.

2. The pyramid of Cheops, shown in the picture, stands on a square base 746 ft. on a side. What is the area of the base? The altitude of this pyramid is 480 ft. Find its volume.



3. What kind of figures are the lateral sides of a pyramid? How is the area of a triangle computed? Find the lateral area (surface, excluding the base) of the pyramid of Cheops (altitude of side 608 ft.).

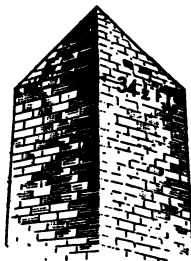
4. Find the numbers to fill the blanks in the following:

ALTITUDE OF PYRAMID.	BASE (Rectangular).	VOLUME.
10 ft.	4 ft. by 5 ft.	()
16 yd.	() sq. yd.	1,728 cu. yd.
()	10 in. by 9.5 in.	1,187.5 cu. in.
5½ in.	16 sq. in.	()
75 ft.	51 sq. ft.	()

5. A pyramid is 50 ft. square and 75 ft. high. Find its volume.

6. The Washington monument is crowned with a square pyramid as shown in the picture. The top of the pyramid is 25 ft. above its base. Find its volume.

7. Find the altitude of a pyramid of which the volume is 84 cu. ft. and the base is a square 6 ft. on a side.



CONES

218. PREPARATORY.

1. Draw a circle of 3 in. radius. Cut along the circle and along a diameter.

2. Roll one of the pieces of paper into a form like the ice-cream mold and place it upright on a sheet of paper. Such a figure is called a **cone**.



3. Mark around the base of the cone. What is the figure thus made?

4. Measure the diameter of this circle. Compute its circumference.

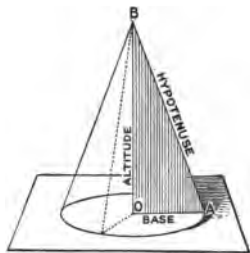
5. Find the area of the sector which formed the cone.

6. Cut from cardboard a right-angled triangle having one perpendicular side longer than the other.

7. Set the triangle upright on a piece of paper, as shown in the picture, and rotate it about the vertical line O B.

8. What figure does the point A trace on the paper?

9. As the triangle rotates about B O, what kind of figure does the hypotenuse generate?



219. The length of the hypotenuse is called the **slant height** of the cone. The point of the cone is called its **vertex**.

220. *The lateral area of a cone is $\frac{1}{2}$ the product of the perimeter of its base and the slant height.*

EXAMPLE : Find the lateral area of a cone having a circumference of 12 in. and a slant height of 3 in.

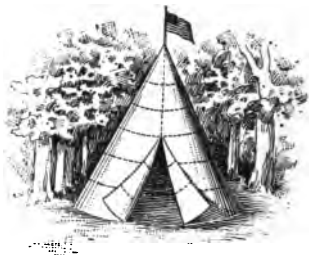
Solution : $\frac{1}{2} \times 3 \times 12$ sq. in. = 18 sq. in.

WRITTEN EXERCISES

1. How many square inches of material are there in the crown of this hat, if the band is 18 in. long?



2. Roy and Harold made a tent for camping. The radius of the base was 6 ft., and the slant height 16 ft. How many square feet were there in its surface?



3. How many square yards of duck were needed to make this tent? What did it cost at $12\frac{1}{2}$ cts. a yard, allowing $\frac{1}{10}$ for waste?

4. What is the area of the surface of a cone made from a sector that is $\frac{2}{3}$ of a circle of radius 6 in.? Also one made from a sector that is $\frac{3}{4}$ of the same circle?

221. PREPARATORY.

1. Jennie wrapped a piece of paper about a cone (Fig. 1) and about a cylinder (Fig. 2) having base and altitude equal



Fig. 1.

Fig. 2.

to those of the cone. What was the shape of the piece of paper in each case?

She rolled the sector into a cone and the rectangle into a cylinder and found that three cone-fuls of sand just filled the cylinder.

2. The volume of the cylinder was how many times that of the cone? The volume of the cone was what part of that of the cylinder?

3. How did their bases and altitudes compare? How is the volume of a cylinder computed?

4. How may the volume of a cone be computed?

222. *The volume of a cone is $\frac{1}{3}$ of the product of the altitude and the area of the base.*

WRITTEN EXERCISES

Find the numbers to fill the blanks :

	1.	2.	3.	4.
Altitude	3 in.	6 ft.	()	3 yd.
Base	12 sq. in.	40 sq. ft.	12 sq. ft.	()
Volume	()	()	144 cu. ft.	96 cu. yd.

SPHERES

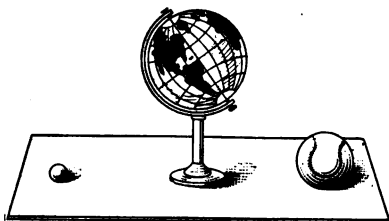
223. If a circle be turned about a diameter as an axis, the surface described is called a **sphere**. The center of the circle is the **center** of the sphere. The space inclosed is called the **volume** of the sphere.

224. The straight line connecting any point of the sphere with the center is called a **radius** of the sphere.

225. A straight line passing through the center of a sphere and terminated by the surface is a **diameter** of the sphere.

Marbles, balls, and globes are examples of spherical forms.

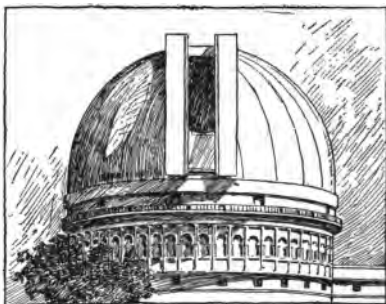
226. When a flat surface or plane cuts a sphere, the section made is a circle. When the cutting plane



passes through the center of the sphere, the circle is called a **great circle**; otherwise, it is called a **small circle**.

Such circles may be seen by immersing some round object, as an orange or a croquet ball, in water to various depths, also by actually cutting spherical bodies, for example, balls of clay.

227. The two portions into which a sphere is divided by any cutting plane are called **segments**. When the cutting plane passes through the center, the segments are equal and are called **hemispheres**.



What is the shape of the dome of this observatory?

228. PREPARATORY.

1. By winding a hemisphere closely with a string and then winding this string on the base of the hemisphere, it is found



that twice as much string is required to cover the convex surface as to cover the base.

Call the area of the base 1. How does the area of the hemispherical surface compare with this area? How does the area of the sphere compare with it?

2. How does the radius of a sphere compare with that of one of its great circles? What represents the area of a circle of radius r ? Then what does $4\pi r^2$ express?

229. *The area of a spherical surface is 4π times the square of its radius.*

WRITTEN EXERCISES

1. Using the letter π , state the area of a sphere of radius: 3 in.; 6 in.; 4 ft.; 1 yd.

Thus, for radius 3 in. the area is 36π .

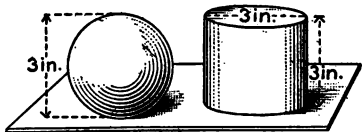
2. Using 3.14 as the value of π , compute the area of each sphere of Exercise 1.

3. The radius of the base of the hemispherical dome of an observatory is 45 ft. Find the perimeter of the base.

4. What is the cost of painting the outer surface of the dome at 2 cts. a square foot, regarding it as a hemisphere?

230. PREPARATORY.

1. Louise modeled from clay a cylinder and a sphere with the dimensions given. The clay cylinder weighed 30 oz. and the sphere 20 oz. The weight of the sphere was what part of that of the cylinder?



2. Find the volume of a cylinder whose base has a diameter $2r$ and whose altitude is $2r$.

3. According to Exercise 2 what part of this volume is the volume of a sphere of diameter $2r$? What represents the volume of the sphere?

231. *The volume of any sphere of radius r is $\frac{4}{3}$ of the radius r times the surface $4\pi r^2$.*

WRITTEN EXERCISES

1. What is the value of $\frac{4}{3}\pi r^3$, taking $r = 8$ and $\pi = 3.1416$?
2. What is the volume of a sphere whose radius is 7 in.?
3. Find the volume of a sphere of radius: 3 ft.; 1 yd.; 2 in.; 4 in.; 5 ft.

Fill the blanks:

For any sphere, let d = diameter, a = area,
 r = radius, v = volume.

	4.	5.	6.	7.	8.	9.	10.	11.
$r =$	()	2	()	()	$\frac{1}{3}$	()	$1\frac{2}{10}$	()
$d =$	()	()	()	8	()	$\frac{4}{10}$	()	$\frac{6}{10}$
$a =$	4π	()	36π	()	()	()	()	()
$v =$	()	()	()	()	()	()	()	()

Solve:

In the following problems use $\pi = \frac{22}{7}$, and simplify by canceling whenever possible.

12. Regarding the earth as a sphere of radius 4,000 mi., find its volume.

13. The gilding of a ball on a church spire cost \$126.72 at \$1.12 per square foot. Find the diameter of the ball.

14. The inner surface of the dome of the capitol at Washington is covered by a fresco painting. Regarding the surface as a hemisphere 165 ft. in diameter, how many square feet are there in the painting?

15. A balloon has the form of a sphere 10 yd. in diameter. How many square yards of material are needed to make the balloon?

16. What is the cost of inflating the balloon with gas costing \$1.40 per 1000 cu. ft.?

17. A kettle is in the form of a hemisphere 28 in. in diameter. How many cubic inches of water does it hold? How many gallons?

18. On a church spire there is a ball 2 ft. in diameter. What will it cost to gild the ball at \$1.20 per square foot?

19. What represents the volume of a hemisphere of radius r ? What represents the volume of $\frac{1}{8}$ of a sphere of radius r ?

20. Express the volume of a sphere whose diameter is d . What is the volume when $d = 15$ in.?

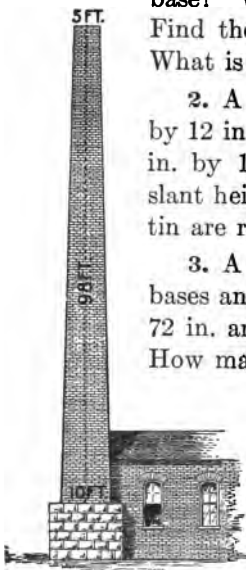
FRUSTUMS

232. When a pyramid is cut parallel to the base, a smaller pyramid is cut off. What is left is called a **frustum of a pyramid**.

Frustums of pyramids are common in such forms as are shown below.

WRITTEN EXERCISES

1. The chimney in the figure is a frustum of a square pyramid with the dimensions marked. What is the area of each base? What kind of figures are the lateral faces? Find the area of one of them. Of all of them. What is the total surface of the frustum?



2. A tin pan measures 8 in. by 12 in. on the bottom and 10 in. by 15 in. on the top; the slant height is 6 in. How many square inches of tin are required to make it?



3. A waste basket has regular hexagons as bases and measures 54 in. around the bottom and 72 in. around the top; the slant height is 17 in. How many yards of silk 24 in. wide are required to line its inner sides, excluding the bottom, and allowing $\frac{1}{2}$ yd. for waste?

4. A manufacturer wishes to purchase cardboard to make 10,000 lampshades; the shades are frustums of regular hexagonal pyramids and measure 15 in. around the top and 36 in. around the bottom with a slant height of 6 in.; he buys cardboard 19 in. by 24 in. by the ream of 480 sheets and finds that 10% of each sheet is wasted in cutting. How many reams must he buy?



233. By cutting a cone parallel to the base, a smaller cone is cut off. What is left is called a **frustum of a cone**.



The objects here pictured are frustums of cones.

If C = convex surface,
 S = slant height,
 A = altitude,

R = radius of lower base,
 r = radius of upper base,
 V = volume,

it can be proved that:

$$1. C = (R + r) \pi S.$$

$$2. V = \frac{\pi A (R^2 + r^2 + Rr)}{3}.$$

These formulas should be used in solving the following exercises. Use $\frac{22}{7}$ for π and simplify by canceling as much as possible.

WRITTEN EXERCISES

1. The bottom of a dish pan is 10 in. in diameter, the top is 14 in. in diameter, and the slant height of the pan 9 in. How many square inches of tin are required to make it?

2. A tub is 26 in. in diameter at the bottom, 40 in. at the top, and 24 in. high, inside measure. How many gallons does it hold?

3. The slant height of the same tub is 25 in., inside measure. How many square feet of lumber are there in it, disregarding the thickness of the lumber?

4. A lighthouse tower is in the shape of a frustum of a cone 20 ft. in diameter at the base, $10\frac{1}{2}$ ft. at the top, and 45 ft. high. The hollow portion within is a cylinder 8 ft. in diameter. How many cubic feet of masonry in the tower?

5. The slant height of the same tower is 45 ft. 3 in. What will it cost to paint the walls outside and inside at $3\frac{1}{2}$ cts. a square foot?

6. The tank of a street sprinkler is in the form of a frustum of a cone with inside diameters 8 ft. at the bottom and 6 ft. at the top, and inside height 5 ft. Find its capacity in cubic feet.

REVIEW

1. If a circle is drawn in a square, as shown in Figure 1, what is the area of the circle?

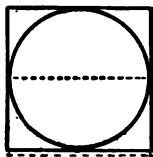


Fig. 1.

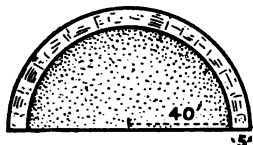


Fig. 2.

2. What is the area of the part of the square not included in the circle?

3. Figure 2 shows a walk about a semicircular garden. Find the length of the outside of the walk. Of the inside.

4. Find the area of the garden and the walk. Of the garden alone. Of the walk.

Find the missing numbers in this table of circles :

	DIAMETER.	RADIUS.	CIRCUMFERENCE.	AREA.
5.	17 in.	()	()	()
6.	()	3.5 ft.	()	()
7.	()	()	()	3.14 sq. ft.
8.	()	()	18.84 ft.	()

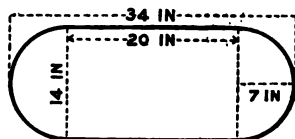
Find the missing numbers relating to cylinders :

	DIAMETER.	ALTITUDE.	LATERAL AREA.	TOTAL AREA.	VOLUME.
9.	10 in.	12 in.	()	()	()
10.	6 in.	()	150.72 sq. in.	()	()
11.	()	8 ft.	()	()	226.08 cu. ft.
12.	()	10 in.	62.8 sq. in.	()	31.4 cu. in.

Solve :

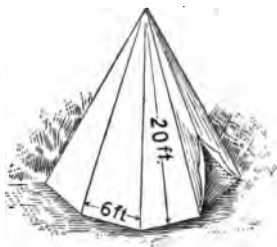
13. An assembly hall is in the form of a rectangle with one semicircular end; the rectangle is 20 ft. by 40 ft. What is the cost of boards for the floor at \$18.20 a thousand feet?

14. A wash boiler is 16 in. deep and has the base indicated in the figure. What is its capacity?



15. What is the radius of a circle whose area is 3.14 sq. ft.?

16. A steam gauge indicates the pressure on a square inch. When the steam gauge reads 45 lb. it means that the steam pressure is 45 lb. on every square inch of inner surface of the boiler. At this rate what is the entire pressure in tons on a cylindrical boiler 2 ft. in diameter and 8 ft. long?



17. An 8-sided tent has the dimensions indicated. How many square yards of canvas are required for it?
18. If the perimeter of the base of a rectangular solid is 16 and its altitude is 5, what is its lateral surface?
19. If the lateral surface is 120 and the altitude 6, what is the perimeter of the solid?
20. If the perimeter of the base of a right pyramid is 20 in. and the slant height is 18 in., what is the lateral surface?
21. What is the perimeter of the base, if the lateral surface is 200 sq. ft. and the slant height 20 ft.?

Fill out the blanks for cylinders:

	22.	23.	24.	25.	26.	27.	28.
Lateral area:	4	()	180	()	$\frac{15}{4}$	$2\frac{1}{2}$	$12m^2$
Altitude:	7	9	()	$\frac{1}{2}$	$\frac{3}{4}$	()	()
Perimeter of base:	()	20	15	$\frac{1}{4}$	()	$1\frac{2}{3}$	$3m$

Fill out the blanks for pyramids:

	29.	30.	31.	32.	33.	34.	35.
Slant height:	28	()	16	75	()	3.5	()
Lateral surface:	()	400	320	375	625	()	$25d^2$
Perimeter of base:	30	40	()	()	15	8.2	$5d$

GENERAL REVIEW

1. Write in words:

2,100,625; 120,865,407; 6,732.086; 15,905.003005.

2. Write, using exponents for powers of 10:

9,000,000; 19,000,000,000.

Add and test:

3. 463281

732008

496207

553484

236729

808909

111222

367428

444333

222555

4. 7632845

4297637

2330099

4280691

2336417

1234567

5432098

6789026

9998885

6767676

5. 96.514

109.009

136.705

297.894

633.946

525.893

600.009

200.946

111.222

333.444

Subtract and test:

6. 893645

598764

7. 208763

145679

8. 106.00897

24.36899

Multiply:

9. 63485×363 .

10. 46390×879 .

11. 49.0076×129.7 .

Divide and test:

12. $634389 \div 623$.

13. $548623 \div 1287$.

14. $18.6032 \div 1.763$.

Add:

15. $\frac{2}{3}; \frac{8}{9}; \frac{7}{8}$.

17. $\frac{1}{12}; \frac{9}{10}; \frac{7}{8}$.

19. $\frac{4}{5}; \frac{5}{7}; \frac{8}{15}$.

16. $2\frac{1}{2}; 3\frac{3}{8}; 5\frac{1}{4}$.

18. $12\frac{1}{2}; 9\frac{3}{4}; 1\frac{1}{8}$.

20. $6\frac{1}{3}; 33\frac{2}{3}; 2\frac{8}{9}$.

Subtract:

21. $\frac{9}{34} - \frac{3}{17}$.

23. $\frac{40}{9} - \frac{17}{18}$.

25. $\frac{23}{8} - \frac{4}{5}$.

22. $9\frac{1}{2} - 4\frac{1}{8}$.

24. $12\frac{3}{8} - 9\frac{7}{16}$.

26. $40\frac{3}{8} - 29\frac{3}{4}$.

Multiply:

27. $\frac{4}{5} \times \frac{11}{12}$.

28. $1\frac{1}{2} \times 4\frac{3}{4}$.

29. $\frac{3}{4} \times \frac{1}{4}$.

30. $5\frac{3}{4} \times 6\frac{1}{2}$.

31. $\frac{1}{2} \times \frac{3}{4} \times \frac{8}{9}$.

32. $12\frac{1}{2} \times 33\frac{1}{2}$.

Divide:

33. $\frac{3}{4} \div \frac{7}{16}$.

34. $4\frac{1}{2} \div 3\frac{3}{4}$.

35. $\frac{4}{5} \div \frac{9}{10}$.

36. $5\frac{1}{2} \div 1\frac{1}{2}$.

37. $\frac{3}{11} \div \frac{2}{11}$.

38. $8\frac{9}{10} \div 4\frac{3}{5}$.

Add:

39. 17 da. 15 hr.

183 20

40. 125 mi. 205 ft.

368 735

41. 40 bu. 3 pk.

87 2

Subtract:

42. 53 gal. 1 qt.

19 3

43. 12 yd. 2 ft.

7 2

44. 127 cu. yd. 8 cu. ft.

99 19

Multiply:

45. 17 A. 40 sq. rd.

5

46. 45 gr. 11 doz.

15

47. 6 mi. 5,000 ft.

9

Divide:

48. 9)23 hr. 45 min. 49. 15)226 yd. 1 ft. 50. 121)400 ft. 9 in.

51. Find the amount of the following bill:

Sept. 6	1 dish pan.....	\$0.75	Sept. 8	1 $\frac{1}{2}$ -inch bit.....	\$0.45
	1 basin.....	.20		1 pt. turpentine.....	.10
	1 8-quart pan.....	.14	" 10	oil and turpentine...	.10
	2 oz. putty.....	.08	" 12	1 steamer.....	.25
" 8	1 saw.....	.90		1 screw-driver.....	.18
	1 wrench.....	.45		1 flour sieve.....	.20
	2 oz. 2-inch brads...	.12	" 15	2 asbestos mats.....	.10
	2 doz. wardrobe hooks	.20	" 17	1 letter plate.....	.50
	1 file.....	.10		3 papers tacks.....	.15
	1 pair 8-inch pliers..	.60	" 21	1 chopping knife....	.25
	1 brace.....	.75	" 25	4 Welsbach gas lamps	4.00
	1 plane.....	1.25	" 27	1 box tapers.....	.10
	1 $\frac{1}{4}$ -inch bit.....	.25		1 gas lighter.....	.25

52. Mr. Marsh's grocery bill for November contained items costing the following amounts:

<i>Nov. 1-2</i>	<i>4-8</i>	<i>9-14</i>	<i>15-20</i>	<i>21-25</i>	<i>26-30</i>
\$0.15	\$0.53	\$0.15	\$0.10	\$0.48	\$0.18
.05	.75	.38	.75	.56	.30
.15	.25	.22	.30	.12	.12
.44	.22	.15	.26	.15	.20
.15	.10	.10	.15	.08	.28
.08	.35	.14	.13	.10	.20
.15	.10	.25	.18	.20	.10
.24	.10	.30	.24	.05	.10
.30	.15	.24	.06	.10	.09
.25	.22	.35	.09	.35	.15

Find the total of the bill.

53. How many quarts in 4 bu. 3 pk. 7 qt.? In 10 bu. 1 pk. 5 qt.?

54. How many square inches in 1 sq. yd.? 5 sq. ft.? 100 sq. ft.?

55. How many yards, feet, and inches in 1,620 in.? In 2,000 in.?

56. How many days, hours, and minutes in 8,256 min.? In 12,000 min.?

Express decimally to the nearest thousandth:

57. $\frac{4}{5}$ ft. 59. $\frac{7}{12}$ yd. 61. $\frac{1}{4}$ gal. 63. $\frac{23}{100}$ mi.

58. $\frac{14}{125}$ bu. 60. $\frac{9}{32}$ hr. 62. $1\frac{1}{2}$ da. 64. $9\frac{1}{2}$ cu. yd.

65. Express decimally as days: 3 da. 19 hr. 40 min.

66. Express decimally as miles: 5 mi. 80 yd. 2 ft.

67. Express 7.65 gal. as gallons, quarts, and pints.

68. A chimney 115 yd. high is built at the rate of 2 yd. 1 ft. 8 in. per day. How many days are required to build it?

69. Find the cost of 6 sacks of flour at 99 cts. a sack. Of 12 shirts at 99 cts. each. Of 7 dozen handkerchiefs at \$1.99 a dozen.

70. What is the area of a floor 16 ft. by 25 ft.? What did it cost at 25 cts. a square foot?

71. 100 oranges sold for \$1.50. Find the rate per dozen.

72. 25 cans of corn sold for \$3.75. Find the rate per dozen.

Find the cost of:

73. 27 lb. sugar at $5\frac{1}{4}$ cts. a lb.

74. 38 lb. steak at 19 cts. a lb.

75. 176 lb. coffee at $33\frac{1}{3}$ cts. a lb.

76. 15 lb. 4 oz. tea at 80 cts. a lb.

77. 197 lb. crackers at $12\frac{1}{2}$ cts. a lb.

78. What is the cost of $\frac{1}{4}$ T. of range coal at \$7.25 a ton, and 100 lb. of cannel coal at \$8 a ton?

79. A butcher sold a lamb as follows: leg, 22.2 lb. at 14 cts. loin, 17.5 lb. at 11 cts.; rib, 14.5 lb. at 10 cts.; chuck, 19.8 lb at 4 cts. What did he receive altogether?

80. What is the cost of 12 chairs at \$1.12 $\frac{1}{2}$ each?

81. What is the cost of 8 books at \$.87 $\frac{1}{2}$ each?

82. How many flags at \$15 each can be bought for \$285?

83. How many wagons at \$87 $\frac{1}{2}$ each can be bought for \$2,450?

84. In a certain year the ten leading butter states of the United States produced the following amounts:

STATE.	MILLION POUNDS.	STATE.	MILLION POUNDS.
Iowa.....	139	Illinois.....	87
New York.....	115	Minnesota.....	82
Pennsylvania.....	111	Michigan.....	68
Wisconsin.....	107	Kansas.....	60
Ohio.....	88	Indiana.....	55

Represent these amounts graphically to the nearest 10 million, using parallel lines.

85. In a recent year 99,311,678 lb. of crude rubber were imported into the United States; the average price was \$4.47 per pound. What was it worth?

86. Spain yields 32,800 T. of cork annually, valued at \$183 a ton. What is the yield worth?

87. The area of Douglas Park in Chicago is 179.79 A.; it is a rectangle about $\frac{1}{3}$ mi. wide. How long is it in miles? In feet?

88. A man set out geraniums 8 in. apart along a bed 5 yd. 2 ft. 4 in. long. If the end plants stand 4 in. from the edges of the bed, how many plants were required?

89. What is the cost of a hardwood floor for a room 15 ft. by 18 ft. at 90 cts. a square yard?

90. A rug 9 ft. wide covers 12 sq. yd. of floor. How long is the rug?

91. A manufacturer wishes to make 25 dozen tables 44 in. wide and 10 ft. long, with square ends. How many square feet of lumber are required to make the tops?

92. In 1900 there were manufactured in the United States 1,825,769 watch movements. If these were placed side by side on a straight line, find the length of the line, estimating that, on an average, one watch would require 2 in.

First make a rough estimate in miles.

93. The fence around a rectangular corner lot is 279 ft. long. Find the frontage of the lot on both streets together.

94. A boy measured the side of the house with a stick which he thought was a yard long, and reported the house to be 42 ft. long. He afterward found that his stick lacked 2 in. of being a yard long. What was the length of the house?

95. A room 21 ft. by 18 ft. is to be covered with carpet 27 in. wide, the breadths running the long way of the room. The cost of sewing is 3 cts. per yard of seam, the cost of the carpet is \$1.12 $\frac{1}{2}$ a yard, and the cost of laying, $\frac{1}{4}$ ct. a yard. Find the cost of the carpet put down.

96. A farmer put 671 bu. of apples into barrels holding 2 bu. 3 pk. each. How many barrels were required?

97. 4 is what per cent of 32? Of 24? Of 12? Of 100?

98. 5 is 10% of what number? 6 is 6% of what number?

99. A 10% cut in the wages of the Rhode Island cotton-mill operators reduced their weekly earnings \$20,000. How much had they been receiving a week?

100. Wages of 75,000 operatives earning \$10 a week were cut 8%. What was the reduction of their total weekly earnings?

101. Ice-cold water increases 4% of its volume if heated to the boiling point. What is the volume of boiling water which measured 5 gal. 3 qt. 1 pt. when ice-cold?

102. If oak wood is burned, the ashes weigh about 3% of the original weight of the wood. How many pounds of wood are burned in producing 5 lb. of ashes?

103. In a recent year 39,673 applications were made for patents at the Patent Office in Washington; 26,418 were granted. What per cent of the number of applications was this?

104. 1,802 of these applications were made by foreigners. How many per cent of the total were made by Americans?

105. An ocean steamer requires provisions as follows:

20 T. of flour and 125% as many tons of potatoes.

2,000 qt. of milk and cream, and 75% as much ice cream.

35,000 lb. fresh meat; $8\frac{1}{2}\%$ as much fish, and 14% as much salt meat.

How many pounds of fish are required? Of salt meat? How many quarts of ice cream? How many tons of potatoes?

106. The total coal production of the United States in 1903 was 359,421,311 T., an increase of 19% over 1902. How many tons were produced in 1902?

107. The value of the coal produced during a recent year was \$504,190,733, an increase of 38% over the previous year. What was the value of the product of the previous year?

108. 1,000,000 bales of cotton often change owners on the New York Exchange in one day during the winter season. What is the value of such a sale at 12 cts. a pound, allowing 500 lb. to the bale?

109. A cotton gin cleans 1000 lb. of cotton in the time it would take a man to clean 5 lb. The cotton gin does how many times the work of one man?

110. Find the total number of bushels of wheat received in the markets of these cities on the date of the statement. On the same date a year before.

The receipts of wheat were as follows:

	To-day. Bushels.	A year ago. Bushels.
Chicago.....	15,000	136,208
Milwaukee.....	6,160	880
Minneapolis.....	71,200	132,050
Duluth.....	6,953	17,260
St. Louis.....	23,000	28,000
Kansas City.....	18,900	36,000

111. If the wheat was sold at an average price of \$.75 a bushel, what was its value in each case?

112. What was the brokerage for selling each amount at $\frac{1}{8}\%$?

113. Mr. Roe sold 5 shares of a carpet company's stock at \$110 a share. How much was this above the par value of the stock? How much did he gain on 5 shares bought at par?

114. An investor in the stock of a cement company sold ten 100-dollar shares at \$90 each. How much below par was this? If he bought them at par how much did he lose?

115. A broker sold 150 shares of copper stocks at \$125 a share and charged $\frac{1}{8}\%$ brokerage. How much did the seller receive?

116. What did 75 shares of railroad stock at \$110 per share cost a purchaser, with brokerage, $\frac{1}{8}\%$?

117. Mr. White's broker sold 25 shares of railroad stock for him at \$116 a share, $\frac{1}{8}\%$ brokerage. What did Mr. White receive?

118. A man bought 50 shares of Pullman Palace Car stock at 220. If an annual dividend of 8% is paid, how much does he receive a year? What per cent of the amount invested is this?

119. In Galveston, Texas, 11,000,000 cu. yd. of earth were used to build an embankment, costing $18\frac{1}{2}$ cts. per cu. yd. To pay for the improvement the city issued \$1000 bonds. How many were required?

120. A collector was to receive 2% of the first \$5,000 of his receipts and $1\frac{1}{2}$ % of the remainder of his receipts. He turned in \$8,934.20. How much did he collect?

121. A commercial traveler receives a commission of 5% on his sales; on Tuesday his sales amounted to 3 times as much as on Monday; his commission for the two days was \$24. How much were his sales for each day?

122. Four workmen receive together \$546 for a job on which they worked 30, 24, 20, and 17 days, respectively. How much should each one receive?

123. Taking the population of the United States to be 76,000,000, how many bushels of wheat are required to feed the people, allowing $4\frac{2}{3}$ bu. to each inhabitant?

124. The wheat crop of the world in a recent year was 2,226,745,000 bu.; this was only 95% of the demand. How many bushels did the crop lack of supplying the demand?

125. In another year the crop was 2,879,924,000 bu., which was 113% of that year's demand. How many bushels were there in excess of the demand?

126. 47,000,000 A. of wheat were sown in this country in one year. How many bushels of seed were required, allowing $1\frac{1}{2}$ bu. to the acre?

127. 50,000 men are engaged in the Dakotas during a wheat harvest. At \$65 a month, how many dollars does it take to pay them for July and August?

128. In a recent year Minnesota sowed 5,000,000 A. of wheat and harvested 78,000,000 bu. What was the yield per acre?

129. Pennsylvania consumed $\frac{2}{3}$ of her wheat crop of 26,000,000 bu. How many bushels were shipped out of the state?

130. A cargo of wheat on the Great Lakes contained 360 carloads of 700 bu. each. How many bushels was this?

131. How many boatloads of 180,000 bu. each does the elevator in the picture hold? How many carloads?

132. How many times would the annual wheat yield of Minnesota (about 80,000,000 bu.) fill the elevator shown?

133. Recently, Minnesota's wheat crop (see Exercise 132) was $1\frac{1}{3}$ times as large as that of Great Britain, twice that of Australasia, and $6\frac{2}{3}$ times that of Egypt. How many bushels were grown by each?



134. A New England cotton mill uses 15 bales of 500 lb. each a day. How many tons does it use in 300 days?

135. This mill employs 600 hands and has a weekly pay roll of \$2,000. What is the average weekly wage for each employee?

136. A cotton plantation of 600 A. yields 250 lb. of cotton an acre. How many 500-pound bales are produced? What is the value of the cotton at 12 cts. a pound?

137. For each pound of cotton of Exercise 136, $2\frac{1}{2}$ lb. of seed were obtained. How many pounds of cotton seed did the plantation yield?

138. The total wheat crop of the United States in a certain year was 670,000,000 bu., of which 40% was consumed at home. How many bushels were consumed at home? How many were exported?

139. Recently, America produced 769,000,000 bu., or about $\frac{1}{4}$ of the world's production. What was the approximate production of wheat in that year? Europe produced $\frac{1}{4}$ of this amount. How many bushels was this?

140. There were in the same year 550,000,000 bread eaters in the world, consuming $4\frac{1}{2}$ bu. per capita. How many bushels were consumed? Was the wheat crop sufficient to meet the demand?

141. The lower Mississippi basin and Atlantic slope is the greatest cotton producing region in the world; its area is $\frac{1}{4}$ of that of the United States. Taking the latter to be 3,025,600 sq. mi., how many square miles are there in the cotton region?

142. A cotton plantation contained 675 acres. Allowing 250 lb. to the acre and 500 lb. to the bale, how many bales are produced by the plantation?

143. The planter sells his cotton for 10 cts. a pound. Find the value of one bale. Find the value of one season's product from this plantation.

144. 28 million acres of cotton were grown in the United States in 1903; the total crop was 10,758,000 bales. What was the average yield in bales per acre? In pounds per acre?

145. What was the total value of this product at 12 cts. a pound?

146. Nearly all of the raw cotton grown in the United States is shipped to the northern states or to Europe, for manufacture. 6,482,849 bales were exported to Europe in 1903. According to Exercise 144 how many bales were retained at home? What per cent (within 1%)?

147. Refined oil is exported in 5-gallon cans. A certain factory for making cans has a daily capacity of 70,000 cans. How many gallons of oil can be shipped in the cans that it makes in one year (300 days)?

148. In 1880 two men could solder 1000 cans in a day; by use of the latest machinery, three men can now solder 24,000 cans in a day. Taking the wages to be the same, the cost of soldering a can now is what part of the cost in 1880?

149. In a recent year the Standard Oil Company imported 60,000 tons of tin, which cost \$10,000,000. How much was this a pound?

150. In 1872 the United States exported 16,363,975 gal. of crude oil; in a recent year it exported 134,892,120 gal. What was the percentage of increase?

151. The world's production of petroleum in one year was recently 5,000,000,000 gallons, of which the United States produced $2\frac{1}{2}$ billions and Russia $2\frac{1}{2}$ billions. How many gallons were produced by all other countries?

152. In a certain leap year there was built on an average 1 oil tank a day; the tanks averaged in capacity 30,000 bbl. What was the total tankage added in this year?

153. The average cost of the tanks mentioned in Exercise 152 was \$8,500 each. What did these tanks cost?

154. What is the value of 1,500 tons of steel rails at \$28 a ton?

155. The following list shows the annual capacities of the leading steel manufactories. Find the total capacity:

Carnegie	2,000,000 tons	National	2,580,000 tons
Federal Steel . . .	2,310,000 "	Am. Steel and Wire	935,000 "

156. In 1890 the value of the steel exports of this country was \$27,000,000; their value in 1900 had increased 480%. What was this value? In 1897 the value was \$62,737,000. What was the percentage of increase from 1897 to 1900?

157. The area of the United States is 3,029,600 sq. mi. The coal fields underlie $\frac{1}{8}$ of the country. How many square miles of coal fields are there?

158. In a recent year, 258,372 T. of bituminous, or soft, coal were mined in the United States, taken from the following regions:

Appalachian field.....	65%	Western field.....	12%
Central field.....	15%	Other fields.....	8%

How many tons were produced by each?

159. When goods listed at \$45 are purchased at a discount of $\frac{1}{3}$, and 20%, what is their cost?

What is the interest on :

160. \$500 at $3\frac{1}{2}\%$ for 1 yr. 8 mo. 6 da.?

161. \$625 at 4% for 2 yr. 10 mo. 15 da.?

162. \$1200 at $2\frac{1}{2}\%$ for 5 yr. 6 mo. 28 da.?

163. \$985 at 3% for 4 yr. 0 mo. 17 da.?

164. \$725 at $4\frac{1}{2}\%$ for 5 yr. 3 mo. 0 da.?

Fill out the blanks for cylinders :

	165.	166.	167.	168.	169.	170.
Volume:	()	()	()	200	640	800
Altitude:	12	20	30	20	16	()
Area of base:	16	25	14	()	()	25

Determine the numbers to fill the blanks :

VOLUME OF PYRAMID.	AREA OF BASE.	ALTITUDE.
171. 432 cu. ft.	()	12 ft.
172. 96 cu. in.	36 sq. in.	()
173. ()	225 sq. ft.	15 ft.

174. What is the volume of a cone whose base contains 18 sq. ft. and whose altitude is 27 ft.?

175. How is the lateral area of a cone found? What is the lateral area of a cone of radius 6 in. and slant height 20 in.?

176. Find the area of the surface of a sphere of radius 3 in.

SUPPLEMENT

THE METRIC SYSTEM

1. The metric system of measures is called international because it has been adopted by the following 43 governments: *

Argentina	France	Nicaragua
Austria-Hungary and territories	French Colonies (including Madagascar)	Norway and Sweden
Belgium	Germany	Ottoman Empire
Bolivia	Greece	Peru
Brazil	Guatemala	Philippines
Bulgaria	Haiti	Porto Rico
Central America	Holland and dependencies	Portugal, Azores, and Madeira
China (28 ports)	Honduras	Roumania
Chili	Italy and dependencies	Russia
Colombia	Japan	Salvador
Costa Rica	Java	San Domingo
Cuba	Mauritius and dependencies	Servia
Ecuador	Mexico	Spain and colonies
Egypt		Switzerland
Finland		Uruguay

In the United States and Great Britain the use of the system is permitted by law, but not made compulsory.

It has not been recognized by law in Morocco, Denmark, Persia, Siam, the interior of China, and among the African races.

Although the international system of measures has not yet been adopted by the United States, our large and growing international commerce requires acquaintance with it. The system is already used to a large extent in the United States by manufacturers for export and by scientists.

* The statements of this page and a few of the exercises following are taken from the report of the Committee on Coinage, Weights and Measures of the 57th Congress.

2. A unit of each denomination is always 10 times as large as the unit of the next lower denomination. This makes the computations conform to the scale of 10, and hence the system is often called the *decimal* system.

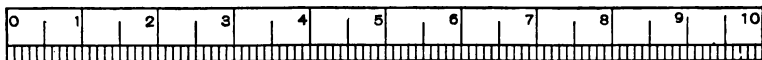
Linear Measure

3. The common units are the **meter**, the **centimeter**, the **millimeter**, and the **kilometer**.

The meter is 39.37 in. long and is used in quite the same way as the yard is used in this country.

LINEAR MEASURE

10 millimeters	= 1 centimeter	10 meters	= 1 dekameter
10 centimeters	= 1 decimeter	10 dekameters	= 1 hektometer
10 decimeters	= 1 meter	10 hektometers	= 1 kilometer



1 DECIMETER.

The following are *standard* abbreviations:

mm. for millimeter	m. for meter	dm. for decimeter
cm. " centimeter	km. " kilometer	

TABLE OF EQUIVALENTS

1 yard = .91 meters	1 meter = 39.37 inches
1 mile = 1.61 kilometers	1 kilometer = .62 miles

Only one unit is commonly used to express a magnitude. For small lengths the millimeter is used. A board would be described as 23 millimeters thick, not 2 centimeters 3 millimeters, just as we say a 10-qt. bucket, not a 2-gal. 2-qt. bucket. For greater lengths, centimeters, meters, and kilometers are the usual units. Thus:

36 cm.	is read 36 centimeters,
7.32 m.	" " 7 and 32 hundredths meters,
56.47 km.	" " 56 and 47 hundredths kilometers.

ORAL EXERCISES

Read the following :

1. 3.5 m.; 84 mm.; 27 cm.
2. A board is 18 mm. thick.
3. A picture is 27 cm. by 32 cm.
4. A remnant of silk contains 3.27 m.
5. From Paris to Calais is 295.32 km.
6. A foot is 30.5 cm. long.

WRITTEN EXERCISES

1. What is the cost of 7 m. of cloth at \$1.80 a meter?
2. What is the cost of 9.2 m. of ribbon at \$.35 a meter?
3. What is the cost of 17.46 m. of lace at \$.55 a meter?
4. How many square meters of carpet will it take to cover a floor 6 m. long and 5 m. wide? One 7.2 m. long and 3.45 m. wide?
5. A rope is 5.36 m. long. How many cm. long is it?
6. How many cm. are there in 16 m.? In 60 mm.?
7. Which divisions of the scale on the opposite page are centimeters? Which are millimeters?
8. How many rounds are there in a ladder 12 m. long when the rounds are 40 cm. apart?
9. How many meters of ribbon would be required to make 35 badges 20 cm. in length?
10. If the current of a river flows 2,160 m. an hour, how many meters does it flow in a minute?
11. 1 meter = () decimeters? 1 m. = () cm.?
12. 1 cm. = () mm.? 1 dm. = () mm.? 1 mm. = .001 of a ()? 1 cm. = .01 of a ()? 1 dm. = .1 of a ()?
13. What are the common international units of length?
14. Express a decimeter in inches. A millimeter.

15. The United States postal service accepts packages not over 30 cm. in length, 20 cm. in width, and 10 cm. in thickness. Express the dimensions in inches.

16. Express the following barometric readings in centimeters: 29.8 in., 30.1 in., 28.6 in.

17. Taking 1 m. = $3\frac{1}{4}$ ft., how many meters in a mile (to nearest meter)?

18. The distance from Paris to Rouen is 136 km. Express this distance in miles (to nearest mile).

Square Measure

4. The common units are the **square meter** (sq. m.), **square centimeter** (sq. cm.), the **square millimeter** (sq. mm.), and the **square dekameter**, the **square hektometer**, and the **square kilometer** (sq. km.).

The square dekameter and the square hektometer when used to measure land are called, respectively, the **are** and the **hektare** (ha.).

SQUARE MEASURE

100 square millimeters	- 1 square centimeter
100 square centimeters	- 1 square decimeter
100 square decimeters	- 1 square meter
100 square meters	- 1 square dekameter
100 square dekameters	- 1 square hektometer
100 square hektometers	- 1 square kilometer

TABLE OF EQUIVALENTS

1 square yard	- .84 square meters	1 square meter	- 1.2 square yards
1 acre	- .405 hektares	1 hektare	- 2.47 acres

WRITTEN EXERCISES

1. How many square millimeters are there in 2 sq. cm.? How many square centimeters are there in 4 sq. m.? How many square millimeters in 1 sq. m.?

2. How many square meters are there in 1 are? How many ares are there in 2 ha.?
3. How many hektares are there in a square kilometer?
4. If a square centimeter of gold leaf is worth 3 cts., what is the cost of 10 sq. m.?
5. 30 ha. are about how many acres? 50 hektares are about how many acres?
6. How many square meters are there in the surface of a street 20 m. wide and 5 km. long?
7. A floor is 5.2 m. by 8.4 m. How many tiles 7 cm. by 13 cm. would be required to pave it?
8. A lot contains 2.7 ares; the owner refused an offer of \$1,800 an are for it; afterward he sold it for \$20 a square meter. Did he gain or lose, and how much?
9. The distance from Paris to Orleans is 121 km.; the railway is a double track, and the rails are 4 m. long. How many rails are used?
10. The roadbed of the railway just mentioned is 15 m. wide. How many hektares does it contain?

Cubic Measure

5. The cubes of the various linear units are used as measures of volume. The most important are the **cubic meter** (cu. m.), the **cubic decimeter** (cu. dm.), the **cubic centimeter** (cu. cm.), and the **cubic millimeter** (cu. mm.).

CUBIC MEASURE

1000 cubic millimeters	- 1 cubic centimeter
1000 cubic centimeters	- 1 cubic decimeter
1000 cubic decimeters	- 1 cubic meter
1000 cubic meters	- 1 cubic dekameter
1000 cubic dekameters	- 1 cubic hektometer
1000 cubic hektometers	- 1 cubic kilometer

The cubic meter is called a **stere** when used in measuring wood.

TABLE OF EQUIVALENTS

1 cubic in. = 16.4 cubic centimeters	1 cubic centimeter = .61 cubic in.
1 cubic ft. = 28.3 cubic decimeters	1 cubic decimeter = .45 cubic ft.
1 cubic yd. = .76 cubic meters	1 cubic meter = 1.31 cubic yd.

WRITTEN EXERCISES

1. If a cubic meter of granite weighs 7 metric tons, what is the weight of a block 5.5 m. long, 3.75 m. wide, and 2.4 m. thick?

2. How many cubic meters of earth are there in an excavation 4 m. long, 6.5 m. wide, and 2.2 m. deep?

3. How many wagon loads of earth of 1 cu. m. each are removed in excavating a cellar 5 m. long, 4.5 m. wide, and 2.8 m. deep?

4. How many cubic centimeters are there in a package 30 cm. by 20 cm. by 10 cm.?

5. How many steres of wood are there in a pile 10.5 m. long, 1 m. wide, and 2.5 m. high? How many cords?

6. At \$7.50 a cubic meter what is the cost of a bridge pier 10.5 m. wide, 3 m. thick, and 13 m. high?

7. How many cubic meters are there in the capacity of a car 10 m. long, 2.5 m. high, and 2 m. wide? How many cubic feet?

8. On ocean steamers $\frac{1}{4}$ cubic meter is sometimes allowed for a passenger's baggage. A traveler has a box 42 cm. by 48 cm. by 76 cm., a trunk 32 cm. by 80 cm. by 50 cm., a chest 45 cm. by 52 cm. by 62 cm., and a box 30 cm. by 40 cm. by 105 cm. Has he excess baggage? If so, how much? What is the charge for the excess at \$10 a cubic meter?

9. A block of granite was 10.4 ft. long, 3.5 ft. wide, and 2.7 ft. thick. How many meters long, wide, and thick was it? How many cu. m. were there in its volume?

10. How many steres of wood are there in a cord of 4-ft. wood?

Measures of Capacity

6. The liter (l.), hektoliter (hl.), and kiloliter (kl.) are the common units for liquid and dry measures. The capacity of the liter is 1 cubic decimeter.

In countries where the metric system is in use, the liter corresponds to the English quart measure. That is, just as milk is sold in New York City by the quart, it is sold in Paris by the liter.

MEASURES OF CAPACITY

10 centiliters	- 1 deciliter
10 deciliters	- 1 liter
10 liters	- 1 dekaliter
10 dekaliters	- 1 hektoliter
10 hektoliters	- 1 kiloliter

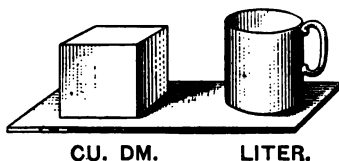


TABLE OF EQUIVALENTS

1 dry quart	- 1.1 liters	1 liter	- .91 dry quart
1 liquid quart	- .95 liter	1 liter	- 1.06 liquid quart
1 bushel	- .35 hektoliter	1 hektoliter	- 2.84 bushels

WRITTEN EXERCISES

1. How many liters of water are there in a tank 3 dm. by 18 dm. by 7.5 dm.?

2. A water tank has a capacity of 1 cu. m. How many liters does it hold?

3. How many hektoliters of grain are there in a bin 4 m. by 5 m. by 2 m.?

4. A grocer bought beans at \$2 a hektoliter and sold them at 3 cts. a liter. What per cent did he gain?

5. A milkman sold 5 kl. of milk in a month at 5 cts. a liter. To how much did the sales amount?

6. A tank 1 m. wide, 5.5 m. long, and 20 cm. deep is full of water. How many liters does it hold? How many quarts?

7. If the rainfall is 11 cm. for a certain month, how many cubic centimeters of water fall on an are of land? How many kiloliters? How many gallons?

8. 2.5 kl. of olive oil was put up in bottles holding .5 l. each. How many bottles were used?

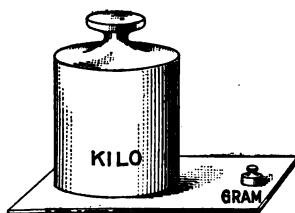
9. A cylindrical tank of radius .52 m. is filled with water to a depth of 2.58 m. How many liters of water are there in the tank? How many quarts in the tank?

10. 2 deciliters are what part of a liter? Of 7 dekaliters? Of 5 hektoliters?

Measures of Weight

7. The weight of a liter of water at a fixed temperature is taken as a unit of weight and is called a **kilogram** or simply a **kilo**. The common units of weight are the **gram** (g.), the **centigram** (cg.), and the **kilogram** (kg.)

The kilo is about 2.2 lb. and is used in the same manner that the pound is used in the United States.



MEASURES OF WEIGHT

10 centigrams	- 1 decigram
10 decigrams	- 1 gram
10 grams	- 1 dekagram
10 dekagrams	- 1 hektogram
10 hektograms	- 1 kilogram
1000 kilograms	- a metric ton - the weight of 1 cu. m. of water.

The weights most commonly used are the kilo and the gram.

TABLE OF EQUIVALENTS

1 pound avoirdupois	- .45 kilogram
1 kilogram	- 2.2 pounds avoirdupois
1 ton	- .91 metric ton
1 metric ton	- 1.1 ton

WRITTEN EXERCISES

1. .5 of a kilogram is how many grams?
2. A cubic decimeter of water weighs how many grams?
How many kilograms?

ONE DECIMETER

1 CU. DM. OF WATER = 1 LITER AND WEIGHS 1 KILO.									
1	2	3	4	5	6	7	8	9	10

3. Alcohol is .83 as heavy as an equal volume of water. A liter of alcohol weighs how many grams? What part of a kilogram? How many pounds?
4. A tank holding 5 cu. m. of water weighs how many metric tons? How many English tons?
5. A tank whose volume is 1 cu. m. contains how many liters? How many kilos does the water weigh? How many pounds?
6. If the weight of ice is .92 of that of water per volume, what is the weight in kilos of a block of ice .2 m. by .2 m. by 5 m.? What is the weight in pounds?
7. Petroleum is .7 as heavy as water per volume. What is the weight in pounds of 1 cu. m. of petroleum?
8. Express 5 centigrams as grams, 3 decigrams as grams, and add the results.
9. Express as kilos, 6 hektograms + 4 dekagrams + 3 grams.
10. Express as hektograms, 9 dekagrams + 5 grams + 8 centigrams.
11. A cubic foot of water weighs 1000 oz. How many grams is this? How many kilograms?
12. A tank 40 ft. long, 10 ft. wide, and 5 ft. deep holds how many cubic feet of water? How many pounds would the water weigh? How many kilograms?

REVIEW

1. If the metric system were used altogether in this country, what points of superiority would it have over the Anglo-American system?

2. A meter being about $1\frac{1}{8}$ yd., how many square yards are there in a square meter?

3. What is the cost of making a cement sidewalk .875 km. long, 3 m. wide at 65 cts. a square meter?

4. 1 cu. dm. of water weighs about 2.2 lb. What is the weight of the water in a tank containing 5 cu. m.?

5. 4 steres make a wagon load of wood. How many loads of wood are there in a pile $10\frac{1}{2}$ m. long, 1 m. wide, and $2\frac{3}{4}$ m. high?

6. The Eiffel tower is 300 m. high. About how many feet high is it?

7. Since a cu. dm. of water weighs 2.2 lb. and granite is 2.7 times as heavy as water, what is the weight of a granite block 2 m. by 3 m. by $5\frac{1}{2}$ m.?

8. Copper is 8.8 times as heavy as water per volume. A cube of copper of edge 10 cm. weighs how many kilograms?

9. How many square meters are there in the floor of a room 6 m. long and 5 m. wide? How many tiles 1 dm. square will it take to cover it?

10. The weight of a liter of hydrogen is .07 of the weight of a liter of air, and 1 liter of air weighs 1.293 g. What is the weight of 1 liter of hydrogen?

11. A cylindrical granite shaft is 2 m. in diameter and 10 m. high. If the stone is 2.7 times as heavy as water per volume, what is the weight of the shaft?

12. The Singer building in New York City is 612 ft. high. How many meters high is it?

THE INTERNATIONAL SYSTEM AND THE ANGLO-AMERICAN
SYSTEM CONTRASTED

1. *Add:* ANGLO-AMERICAN.

YD.	FT.	IN.
12	2	7
9	1	8
14	0	11
6	2	5
7	1	10
51	0	5

Add: INTERNATIONAL.

METERS.
12.27
9.18
14.11
6.25
7.20
49.01

2. *Reduce to inches:*

3 mi.
167 rods
3 yd.
2 ft.
9 in.

Reduce to millimeters:

5 kilometers
6 hektometers
7 dekameters
1 meter
5 decimeters
1 centimeter
2 millimeters

Ans., 223,287 in.

Ans., 5,671,512 mm. (by inspection).

3. Find the contents of a tank:

8 ft. 4 in. long
5 ft. wide
2 ft. 5 in. deep

2.5 m. long
1.52 m. wide
80 cm. deep

Ans., 100 cu. ft. 1,200 cu. in.

Ans., 3.04 cu. m.

4. What is the pressure on the bottom when the tank is filled with water?

Ans., 1.044 lb. per sq. in.

Ans., 80 g. per sq. cm. (No computation needed, since 1 cu. cm. of water weighs 1 g., and the tank is 80 cm. deep.)

5. Find how much the tank holds:

In gallons: (1 gal. = 231 cu. in.).

Ans., 753 $\frac{4}{11}$ gal.

In liters:

Ans., 3,040 l.

6. Find the weight of water filling the above tank.

Ans., 3 tons 264 lb.

Ans., 3.04 metric tons (by inspection).

FOREIGN MONEYS

8. The prices at which foreign money is actually bought and sold will vary slightly from the following official values, fixed by the United States Treasury. The market price is called the **rate of exchange**.

TABLE OF VALUES OF FOREIGN MONEYS

COUNTRY.	MONETARY UNIT.	VALUE IN UNITED STATES GOLD.
Canada.....	Dollar	\$1.00
Great Britain.....	Pound sterling	4.8665
France.....	Franc	.193
Belgium.....	Franc	.193
Switzerland.....	Franc	.193
Italy.....	Lira	.193
Spain.....	Peseta	.193
Greece.....	Drachma	.193
Germany.....	Mark	.238
Austria-Hungary.....	Crown	.203
Cuba.....	Peso	.926
Mexico.....	Dollar (silver)	.443

Currency of Great Britain

1 pound sterling	- 20 shillings	£1 - 20s.
1 shilling	- 12 pence	1s. - 12d.
1 penny	- 4 farthings	1d. - 4 far.

9. The rate of exchange for sterling is usually expressed by stating in United States money the amount for which £1 sterling can be bought or sold.

Thus, "sterling exchange 4.88" means that \$4.88 is paid for £1.

Fractions of the smallest current coin, as the cent of United States money, are to be rejected if less than $\frac{1}{2}$, otherwise to be counted as a whole unit.

WRITTEN EXERCISES

1. When sterling exchange is 4.88, what will a draft on London for £200 cost?

2. If sterling exchange is quoted at 4.87½, how much English money can be procured for \$1000?

3. A merchant owes a London firm £1,421 14s. 6d. (usually written, £1,421-14-6). What must he pay for a draft for this amount at 4.87½?

4. The English Parliament has voted the King an annual income of £470,000. Make an approximate estimate of the equivalent in United States money, taking £1 = \$5.

5. The income of the government of the United Kingdom for a recent year is given in the table. Fill the blank:

Taxes.....	£98,496,000
Post-offices.....	13,192,000
Telegraphs.....	3,390,000
Remainder.....	()
Total.....	£128,633,000

6. Make an approximate estimate of the equivalent of each item of the above table in United States money.

7. Determine the equivalent of each item to the nearest cent in United States money, using £1 = \$4.8665.

8. The savings banks deposits average as follows *per capita* of population in the countries named:

	£	s.	d.		£	s.	d.
Denmark.....	15	11	6½	United States.....	6	4	10½
Switzerland.....	13	0	2½	Austria-Hungary...	5	8	3½
Germany.....	7	10	6½	France.....	4	8	8½
Norway.....	7	8	7½	Great Britain.....	4	2	5½
Australia.....	7	6	4½	Canada.....	2	5	2½

Express each amount in United States money.

9. Express in United States money the items of the following table, and fill the blanks:

GREAT BRITAIN.	IMPORTS FROM.	EXPORTS TO.
British Possessions.....	£99,433,955	£83,426,761
United States.....	126,062,155	14,716,489
France.....	51,396,793	13,706,246
Germany.....	28,534,159	22,525,987
Others.....	()	()
Total.....	£470,378,583	£233,359,987

Currency of France

10. The unit is the **franc** (fr.); it is divided into 100 centimes (pronounced *sontem*). 100 centimes equal 1 franc.

Hence, hundredths of a franc are read centimes.

Thus, 14.25 fr. = 14 francs, 25 centimes; 0.60 fr. = 60 centimes.

It is customary in France and Germany to write a zero before the decimal point when the number is a pure decimal.

11. The price of French money is usually quoted by stating how many francs are worth one dollar.

Thus, if francs are quoted at 5.15½, 5 francs and 15½ centimes are equivalent to \$1. For approximate estimates, 1 franc = 20 cts., 5 francs = \$1.

WRITTEN EXERCISES

Find in francs the total cost of the following purchases :

- 23.64 meters of silk at 7.25 francs per meter.
- 8.5 meters of velvet at 9.95 francs per meter.
- 127.85 meters of ribbon at 0.60 franc per meter.

At 5.15 how many francs can be procured for \$500?

How much will a draft for 2,000 francs cost at 5.15½?

6. A traveler exchanged \$15.80 for francs at 20 cts. per franc. How many francs did he get?

7. A teacher sent to Gauthier, a publisher in Paris, for some books. The bill was 47.45 francs. How many dollars did the teacher remit, exchange being $5.15\frac{1}{2}$?

8. When a franc is worth 19.3 cts., how many dollars will a draft for 455.5 francs cost?

9. How many francs will \$9.55 buy when exchange is quoted at $5.15\frac{1}{2}$?

10. Imports and exports of France in a recent year were:

COUNTRY.	IMPORTS. <i>Million francs.</i>	EXPORTS. <i>Million francs.</i>
British Isles.....	669	1,239
United States.....	460	254
Germany.....	412	460
Others.....	()	()
Total.....	4,409	4,079

In each case what per cent of the total was the item for the United States? Supply the missing numbers.

11. Express in dollars each number in Exercise 10.

Currency of Germany

12. The unit is the Reichsmark or **Mark** (M.). It is divided into 100 Pfennigs (pfgs.). 1 mark = 100 pfgs.

Hence, hundredths of a mark are read pfennigs.

Thus, 4.01 M. = 4 marks and 1 pfennig. 0.70 M. = 70 pfennigs.

13. The rate of exchange for German money is usually stated by naming in cents the equivalent of 4 marks.

Thus, when marks are quoted at $95\frac{1}{2}$, 4 marks are sold for $95\frac{1}{2}$ cts.

For approximate estimates, take one mark as 25 cts.

WRITTEN EXERCISES

Find the total cost in marks of the following purchases :

1. 8.6 meters of calico at 0.45 M. per meter.
2. 14.5 meters of muslin at 0.65 M. per meter.
3. 12 meters of cloth at 5.28 M. per meter.
4. 0.6 kilos of butter at 3.30 M. per kilo.

5. Make approximate estimates of the equivalents in United States money of the prices and of the results of Exercises 1 to 4.

Solve :

6. When marks are quoted at $95\frac{1}{2}$, what will be the cost of a draft on Berlin for 20,000 M.?

7. If marks cost $95\frac{3}{4}$, how many marks can be procured for \$1,200?

8. The following are the expenditures of the German Empire in a recent year:

	MARKS
Army.....	665,850,576
Navy.....	172,207,975
Postal and telegraph.....	355,910,050
Railways.....	79,925,610
Administration.....	<u>586,358,408</u>

Estimate the equivalent amounts of United States money for each item.

9. Compute the amount in dollars equivalent to each number in Exercise 8 when exchange is $95\frac{1}{2}$.

Comparison of United States and Foreign Moneys

14. To find the approximate equivalents of amounts of foreign money in United States money, use the following:

One pound - \$5.00.	One mark (M.) - \$.25.
One shilling - \$.25.	One franc (fr.) - \$.20.

WRITTEN EXERCISES

1. An American in London sees the following prices in the shop windows: hats, 5/9 (5s. 9d.); gloves, 2/11½; suit, 65/0; overcoat, 2-10-6 (£2 10s. 6d.); necktie, 1/5½. What would be the approximate equivalents in United States money?

2. The following purchases were made in London:

2½ yd. lace at 18/6 a yard. 16 yd. silk at 10/3 a yard.

7½ yd. velvet at 13/9 a yard. 50 yd. ribbon at 1/3 a yard.

Find the total cost in English money and find the approximate cost of each item in United States money.

3. Find the exact cost of the whole purchase mentioned in Exercise 2, exchange being quoted at \$4.88½.

4. A man offered a sovereign (1 pound) in payment of a book and received 12s. 6d. in change. What did the book cost?

5. A person imported some books from Berlin; the bill was 200 M. About how many dollars was this?

6. A traveler paid 15.50 fr. for a railway ticket. About how many dollars and cents did the ticket cost?

7. A man's hotel bill in Paris was 67 fr. About how many dollars was this?

8. A man spent 1,300 M. while sightseeing in Germany. About how many dollars was this?

9. A traveler bought an Oriental rug for 240 M. Find the approximate equivalent of the price in United States money. Find the exact equivalent, using 23.8 cts. per mark. Add 40% for duty and find the total cost of the rug.

10. It cost 100,000 fr. to lay out one of the national parks of France. How many dollars was this?

11. A traveler had 215 francs and 90 centimes when he went from Paris to Berlin; he exchanged his money for marks and pfennigs. How many did he receive?

Money Orders

15. Travelers in foreign countries often carry bank or express money orders instead of cash. The following is a bank money order:

No 168		Chicago, Feb. 6, 1909.	
Pay to the order of		Payee's Signature: <i>Daniel A. Peirce</i>	
FIFTY DOLLARS or the equivalent as follows			
AUSTRIA	ENGLAND	FRANCE	GERMANY
HUNGARY	IRELAND	NETHERLANDS	HOLLAND
SCOTLAND	SWITZERLAND	SPAIN	ITALY
CROWN	SHILLING	FRANC	MARK
245. 14.	10. 4. 1.	256. 25.	206. 25.
		122. 57.	258. 25.
		158. 48.	
ALL OTHER COUNTRIES AT CURRENT RATES ON LONDON.			
To the Banks and Bankers mentioned on the back hereof		THE CHICAGO NATIONAL BANK	
		John Doe	

This order cost the purchaser, Mr. Peirce, \$50. The amount of money that he will receive if he presents it at any of the banks in England named on the back of the order is shown on its face to be £10 4s. 1d. Thus, because of the rate of exchange, the traveler does not always receive the equivalent of \$50 when he cashes the order.

WRITTEN EXERCISES

1. When £1 of English money is worth \$4.866, what are £10 4s. 1d. worth?
2. The cash value of the above order in Germany is 206.25 marks. At 23.8 cts. a mark, find the equivalent of this amount in United States money.
3. The cash value of the same order in France is 256.25 francs. At 19.3 cts. a franc, express this amount in dollars.
4. Find in English money the cash value of 10 fifty-dollar express money orders at the rate given in the above order. Find similarly their cash value in Paris. In Berlin.
5. Find in United States money the equivalent of the order shown above, if cashed in Austria, the crown being 20.3 cts.

ALGEBRA

Definitions

16. An expression that does not contain within itself the sign of addition or of subtraction is called a **monomial**.

Thus, 7 , $\frac{26}{5}$, $\sqrt{18}$, a , a^2c , $\frac{ab}{4^2}$, $xy \div a$ are monomials.

17. Two monomials connected by the sign $+$ or the sign $-$ are called a **binomial**.

Thus, $7 - 4$, $8 + 6$, $a + b$, and $c - 5d^2$ are binomials.

18. Three monomials connected in any way by addition or subtraction signs are called a **trinomial**.

Thus, $axy - 2bc + 4mn$ is a trinomial.

19. The numbers separated by the sign $+$ or $-$ are called **terms** of the expression.

Thus, $6 + 8 + 4$, $6 + 8 - 4$, $12 - 7 - 3$, $a + 5b - c$ are trinomials. 6 , 8 , and 4 are the terms of $6 + 8 + 4$; and a , $5b$, c are the terms of $a + 5b - c$.

20. The **parenthesis**, $()$, may be used to group several terms into one expression. The absence of a sign between a number and a parenthesis indicates multiplication.

The following examples illustrate the method of reading expressions containing parentheses:

$4(8 - 3)$	is read "4 times the binomial, $8 - 3$."
$a(b - c + d)$	" " "a times the trinomial, $b - c + d$."
$(a + b)(4 - 3)$	" " "The binomial, $a + b$, times the binomial, $4 - 3$."
$(8 + 6 - 2) \div 4$	" " "The trinomial, $8 + 6 - 2$, divided by 4."
$(b + 6)^2$	" " "The square of the binomial, $b + 6$."
$(a - b + c)^2$	" " "The cube of the binomial, $a - b + c$."

ORAL EXERCISES

Read the following:

- | | |
|------------------------------------|----------------------|
| 1. $(17 + 15) \div (9 - 3 + 10)$. | 5. $(12 - 7)^2$. |
| 2. $(a + b) - (c - d)$. | 6. $(x + y) - c$. |
| 3. $(a + b - c) + (b - d)$. | 7. $(x + 1)^2$. |
| 4. $(a + b)(b - c)^2$. | 8. $(a^2 + b^2)^2$. |

Select the monomials; the binomials; the trinomials:

- | | | |
|-----------------------|------------------|-----------------------|
| 9. $a + b - c$. | 12. $4x^2 + 7$. | 15. $a + b + c + d$. |
| 10. $x - y + z + w$. | 13. m . | 16. $a + 5b - c$. |
| 11. $2g - 5 + x^2$. | 14. $gt^2 + a$. | 17. $5x^2 + ay^3$. |

Addition and Subtraction

ORAL EXERCISES

1. $3a + 4a = ?$ If a is 1, what is the value of $3a$? Of $4a$? Of $3a + 4a$?

2. $6b + 5b = ?$ If b is 1, what is the value of $6b$? Of $5b$? Of $6b + 5b$?

3. If $a = 1$ and $b = 1$, what is the value of $3a + 6b$? What is the value of $4a + 7b$?

Find the value of each of these expressions, when each letter = 1:

- | | | |
|-------------------|-------------------|----------------------|
| 4. $a + 2b$. | 7. $2a + 2b$. | 10. $3a + b$. |
| 5. $a + b + c$. | 8. $a + 2b + c$. | 11. $2a + 3b + 3c$. |
| 6. $b + c + 2a$. | 9. $b + d + 3c$. | 12. $c + 4d + 2a$. |

13. Find the numbers to fill the blanks:
 What is the sum of $3a + 4b$ and $2a + 6b$? $\begin{array}{r} 3a + 4b \\ 2a + 6b \\ \hline \end{array}$ $\begin{array}{r} 7 \\ 8 \\ \hline 15 \end{array}$

14. When $a = 1$ and $b = 1$, what is the value of each addend in Exercise 13? $()a + ()b$
 What is the sum of these values? What is the value of the sum? Explain how to test the work of adding.

Add and test by letting $a=b=c=1$:

$$\begin{array}{r} 15. \ a + b + c \\ 2a + b + 3c \end{array}$$

$$\begin{array}{r} 16. \ a + 2b + c \\ a + 6c \end{array}$$

$$\begin{array}{r} 17. \ 6a + b + c \\ 2b + c \end{array}$$

18. Find the numbers to fill the blanks in this subtraction.

Let $a = 1$, $b = 1$. How is the work tested?

$$\begin{array}{r} 4a + 7b \quad 11 \\ 2a + 5b \quad 7 \\ \hline ()a + ()b \quad 4 \end{array}$$

WRITTEN EXERCISES

Add and test:

$$\begin{array}{r} 1. \ a + 3b \\ 10a + 11b \end{array}$$

$$\begin{array}{r} 4. \ 5a + 9b \\ 5a + 3b \end{array}$$

$$\begin{array}{r} 7. \ m + n + p \\ 2m + 5n + 9p \end{array}$$

$$\begin{array}{r} 2. \ a + b + c \\ m + 6c \end{array}$$

$$\begin{array}{r} 5. \ 4a + c \\ 2a + b + c \end{array}$$

$$\begin{array}{r} 8. \ 45m + 3n + p \\ 5m + 2n + p \end{array}$$

$$\begin{array}{r} 3. \ 16a + 4b \\ 4b + 16a \end{array}$$

$$\begin{array}{r} 6. \ 6a + b \\ 3a + 2b + c \end{array}$$

$$\begin{array}{r} 9. \ 6m + 4n + p \\ 4m + 6n + 9p \end{array}$$

Subtract and test:

$$\begin{array}{r} 10. \ 6a + 10b \\ 2a + 5b \end{array}$$

$$\begin{array}{r} 14. \ 2a + 5b + 4c \\ a + b + c \end{array}$$

$$\begin{array}{r} 18. \ 3x + 7y + 11z \\ 2x + y + 4z \end{array}$$

$$\begin{array}{r} 11. \ 5a + 6b \\ a + 3b \end{array}$$

$$\begin{array}{r} 15. \ 2a + b + 7c \\ a + b + 5c \end{array}$$

$$\begin{array}{r} 19. \ 45m + 3n + 12p \\ 5m + n + 2p \end{array}$$

$$\begin{array}{r} 12. \ 10a + 5b \\ 9a + 5b \end{array}$$

$$\begin{array}{r} 16. \ 10a + 6b + 5c \\ 5a + b + 5c \end{array}$$

$$\begin{array}{r} 20. \ 16a + 2b + 2c \\ 12a + b \end{array}$$

$$\begin{array}{r} 13. \ 17a + 9b \\ 17a + b \end{array}$$

$$\begin{array}{r} 17. \ 25 + n + 7p \\ 5 + n + p \end{array}$$

$$\begin{array}{r} 21. \ 17m + 6n + 2p \\ 9m + n + 2p \end{array}$$

Solve:

22. A fruit grower picked $2a$ bu. of apples, $3b$ bu. of peaches, and $4c$ bu. of plums in one season. Indicate the sum of these numbers to show how many bushels of fruit he picked.

23. The same man picked $4a$ bu. of apples, b bu. of peaches, and $3c$ bu. of plums the next season. How many bushels of fruit did he pick? How many bushels did he pick in the two seasons?

24. For a house a builder used of three kinds of lumber $50a$ ft., $10b$ ft., $12c$ ft. respectively; for another house he used of the same kinds $75a$ ft., $15b$ ft., and $10c$ ft. How many feet did he use in all?

25. A grain dealer had $8m$ bu. of oats, $10n$ bu. of wheat, and $7p$ bu. of rye, and sold $6m$ bu. of oats, $5n$ bu. of wheat, and $7p$ bu. of rye. How many bushels of grain had he left?

Multiplication

21. The product of two or more letters is indicated by placing them next to each other without a sign of equation.

Thus, the product of a , b , and c , or $a \times b \times c$, is written abc .

22. The number written above and at the right of a number to show how many times it is taken as a factor is called an **exponent**.

Thus, in c^2 , the 2 shows that c^2 means $c \times c$ or cc ; also in 3^2y^3 the 2 and 3 show that $3^2y^3 = 3 \times 3 \times y \times y \times y$.

WRITTEN EXERCISES

Write the equivalent forms by using exponents:

- | | | | |
|-------------------------------------|--------------|---------------------------------------|-----------------|
| 1. 9 (or 3×3). | 5. 25 . | 9. 49 . | 13. 100 . |
| 2. 8 (or $2 \times 2 \times 2$). | 6. 16 . | 10. $sssss$. | 14. $aayyy$. |
| 3. 81 . | 7. bb . | 11. ccc . | 15. $100aabb$. |
| 4. $8aa$. | 8. $16bbb$. | 12. 27 (or $3 \times 3 \times 3$). | |

Indicate by use of exponents the number of:

16. Days in 7 weeks.

19. Seconds in an hour.

17. Things in a gross.

20. Cubic inches in a cubic foot.

18. Months in 12 years.

21. Years in 100 centuries.

Multiply:

$$\begin{array}{r} 22. \quad 4a \\ \quad 3a \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 5b \\ \quad 2b \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 8c^2 \\ \quad 3c \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad 4x^3 \\ \quad 2x^2 \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 5m \\ \quad 8m \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad 6b^2 \\ \quad 3b^2 \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad 4x \\ \quad 2x^2 \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad 40s \\ \quad 3s^2 \\ \hline \end{array}$$

23. The following examples illustrate the process of multiplying literal expressions of more than one term.

$$\begin{array}{r} a + 7b \qquad 8 \\ a + b \qquad \quad 2 \\ \hline a^2 + 7ab \\ \quad ab + 7b^2 \\ \hline a^2 + 8ab + 7b^2 \quad 16 \end{array}$$

$$\begin{array}{r} a + 2 \qquad 3 \\ a + b \qquad \quad 2 \\ \hline a^2 + 2a \\ \quad + ab + 2b \\ \hline a^2 + 2a + ab + 2b \quad 6 \end{array}$$

We multiply the multiplicand by each term of the multiplier, place like terms in column, and add.

The numbers at the right are the values of the corresponding expressions when $a = 1$, $b = 1$. These serve to check the work. Thus, in the first example, $2 \times 8 = 16$; and in the second, $2 \times 3 = 6$.

WRITTEN EXERCISES

Multiply and test the work by giving values to the letters:

$$\begin{array}{r} 1. \quad 3a + x \\ \quad a + b \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 4x + 5 \\ \quad a + x \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 2 + a \\ \quad b + x \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad y + 3 \\ \quad 3y + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad x + 1 \\ \quad x + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad n + 2 \\ \quad n + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad z + 5 \\ \quad z + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad x + y \\ \quad x + y \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad s + 6 \\ \quad s + 3 \\ \hline \end{array}$$

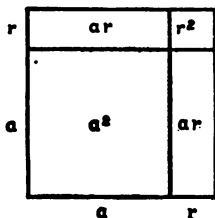
$$\begin{array}{r} 6. \quad 1 + m \\ \quad 2 + m^2 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad a + x \\ \quad b + y \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad x^2 + 2 \\ \quad x + 1 \\ \hline \end{array}$$

Solve:

13. Point out in the figure that the square on $a + r$ is the square on a plus the square on r plus twice the rectangle of a and r .



14. Multiply $a + r$ by $a + r$.

15. Replace a by 30, and r by 2, throughout the work of Exercise 14. What number have you squared?

16. What is the result if $a = 40$, $r = 3$? What is the square of 43?

24. The result of Exercise 14 says: .

The square of the sum of any two numbers is the square of the first plus twice the product of the first and the second, plus the square of the second.

1. Square 17 by writing it $10 + 7$.

2. Find similarly: 24^2 ; 13^2 ; 41^2 ; 25^2 ; 42^2 ; 33^2 .

Multiply:

3. $x + 10$ by $x + 20$.

5. $a + 1$ by $b + 1$.

4. $2a + 1$ by $2b + 3$.

6. $3x + y$ by $3y + x$.

Division

25. In division we have the product and one factor given to find the other factor.

Thus, in $40ax \div 20a$, $40ax$ is the product of $20a$ and another factor, which is evidently $2x$.

ORAL EXERCISES

State the quotients in the following cases of division:

	1.	2.	3.	4.	5.
Dividend:	$36x$	15	$15a$	$14b$	xy
Divisor:	9	5	5	7	y

Find the numbers to fill the blanks :

	6.	7.	8.	9.	10.
Dividend:	()	()	$21ay$	$42t$	$18ab$
Divisor:	$3a$	$9y$	()	$7t$	$9a$
Quotient:	$4b$	$2x$	$7a$	()	()

26. The following example illustrates the process of dividing literal expressions of more than one term.

EXAMPLE: Divide $a^2 + 3ab + 2b^2$ by $a + b$.

$$\begin{array}{r}
 a + 2b \\
 a + b \overline{) a^2 + 3ab + 2b^2} \\
 \underline{a^2 + ab} \\
 2ab + 2b^2 \\
 \underline{2ab + 2b^2} \\
 0
 \end{array}$$

Divide the first term of the dividend by the first term of the divisor; the quotient is a . Multiply the entire divisor by a and subtract, as in the case of numbers. Divide the first term of the remainder by the first term of the divisor again; the quotient is $2b$. Multiply and subtract, as in the case of numbers.

Test: Compare the product of the divisor and quotient with the dividend.

WRITTEN EXERCISES

Divide and test :

- $c^2 + 2cd + d^2$ by $c + d$.
- $mn + np + nq$ by n .
- $mp + p^2 + pq$ by p .
- $4a^2 + 4ab + b^2$ by $2a + b$.
- $a^2 + 4ab + 4b^2$ by $a + 2b$.
- $3a^2 + 7ab + 2b^2$ by $3a + b$.
- $x^2 + 5xy + 6y^2$ by $x + 2y$.
- $m^2 + 3mn + 2n^2$ by $m + n$.

Factors and Roots

27. A product may be given, and it may be required to find the factors; that is, a set of numbers whose product is the given number. This is called **factoring** the given expression. It may usually be done in various ways.

Thus, $28 = 4 \times 7$, or 2×14 , or $2 \times 2 \times 7$, or 28×1 .

Also, $4ax = 4 \times a \times x$, or $4a \times x$, or $a \times 4x$, or $2 \times 2a \times x$, or $4 \times ax$.

28. An integral factor is one that is not in the form of a fraction. By factor is meant integral factor, unless otherwise stated.

ORAL EXERCISES

Find two different sets of factors for each of the following:

- | | | |
|--------------|--------------|-----------------|
| 1. $6a$. | 4. $15axy$. | 7. $7ay$. |
| 2. $6abc$. | 5. $8a^2b$. | 8. $14br^2$. |
| 3. $14a^2$. | 6. $48x$. | 9. $21a^3x^2$. |

Regarding $3a$ as one of two factors, state the other factor of each of the following expressions:

- | | | |
|---------------|----------------|----------------|
| 10. $6a$. | 13. $9ab$. | 16. $30axy$. |
| 11. $3ac$. | 14. $12az$. | 17. $21a^2$. |
| 12. $15a^3$. | 15. $24ay^2$. | 18. $15ax^4$. |

Regarding x as one of two factors, state the other factor of each of the following expressions:

- | | | |
|--------------|---------------|---------------|
| 19. $4x$. | 23. x^2 . | 27. $1x$. |
| 20. $10ax$. | 24. $10axy$. | 28. $6a^2x$. |
| 21. $2bx$. | 25. $5x^2z$. | 29. $7ax^2$. |
| 22. mx . | 26. x^3 . | 30. $2rx^4$. |

29. The symbol $\sqrt{\quad}$ means "the square root of."

Thus, $\sqrt{196} = 14$.

Similarly, $400 = 2^4 \times 5^2$; hence $\sqrt{400}$ is $2^2 \times 5$, or 20.

Also, $169a^2b^4 = 13^2a^2b^4$; hence $\sqrt{169a^2b^4}$ is $13ab^2$.

ORAL EXERCISES

State the square root of:

- | | | |
|---------------|---------------|------------------|
| 1. 25. | 5. a^2 . | 9. b^6 . |
| 2. 144. | 6. 81. | 10. a^2b^2 . |
| 3. $16a^4$. | 7. $49x^2$. | 11. $64b^2$. |
| 4. $121m^4$. | 8. $225c^4$. | 12. $16a^4c^6$. |

30. When there are so many factors in the numerical part that it is not easy to separate them into two equal groups by inspection, the factors may first be written out in full, and then grouped.

$$\begin{aligned}\text{Thus, } 7,056 &= 4 \times 4 \times 3 \times 3 \times 7 \times 7 \\ &= (4 \times 3 \times 7) (4 \times 3 \times 7) = 84^2.\end{aligned}$$

$$\text{Therefore, } \sqrt{7056} = 84.$$

The work of finding the factors may be arranged thus:

$$\begin{array}{r} 4 \overline{)7056} \\ 4 \overline{)1764} \\ 3 \overline{)441} \\ 3 \overline{)147} \\ 7 \overline{)49} \\ 7 \end{array}$$

WRITTEN EXERCISES

- Find: $\sqrt{625}$; $\sqrt{11,025}$; $\sqrt{1024b^8}$.
- Find: $\sqrt{256a^4b^4}$; $\sqrt{3025a^2}$; $\sqrt{169a^4b^6}$.

Find the square root of:

- $625m^2p^4$.
- $256x^2y^2$.
- $144s^8t^6$.
- $4,225m^2n^{10}$.
- $1,225a^6b^6$.
- $361p^8q^8$.
- $196m^6n^4$.
- $441x^2y^{12}$.

Order of Operations

31. When not otherwise indicated, multiplications and divisions are performed before additions and subtractions.

Thus, $3 + 7 \times 2$ is taken to mean $3 + 14$, or 17.

Similarly, $4 + 12 \div 2$ means $4 + 6$, or 10.

$$\text{Also, } 5 + 14 \div 7 - 3 = 5 + 2 - 3 = 7 - 3 = 4.$$

ORAL EXERCISES

Perform the operations indicated:

- $12 - 6 \div 3$.
- $30 + 5 \times 8$.
- $6 \times 4 - 3$.
- $2 + 8 \times 4 - 3$.
- $6 \times 5 - 12 + 7$.
- $4 \times 9 - 10 \div 5$.
- $8 + 2 \times 7$.
- $12p - 16p \div 4p$.
- $40r - 36r \div 2r$.
- $52p \div 26p + 5p$.
- $14x \times 3x - 2x \times 10x$.
- $34y + 6y \div 2y - 17y$.
- $30y + 2y \times y \div 3y$.
- $40s + 2s \div 7s + 4s$.

32. To indicate that additions and subtractions are to be performed first, the parenthesis is used.

Thus, in $(3 + 7) \times 2$, the parenthesis shows that 3 and 7 are first to be added, then the result multiplied by 2. It indicates that what is within the parenthesis is to be treated as a single expression.

Similarly, $(4 + 12) \div 2$ means $16 \div 2 = 8$.

Perform the operations indicated:

- | | |
|-----------------------------------|------------------------------------|
| 1. $(12a - 6a) \div 3a$. | 5. $8b \times (30b + 5b)$. |
| 2. $(2c + 8c) \times (4c - 3c)$. | 6. $6h \times (5h + 7h - 10h)$. |
| 3. $(16 - 12) \div 4$. | 7. $(84x + 6x) \div (22x - 17x)$. |
| 4. $(40c - 36c) \div 2c$. | 8. $(8m + 12m) \div (7m - 2m)$. |

Equations

33. Two equal expressions connected by the sign of equality are called an **equation**.

Thus, $3x + 10 = 40$. $ax + b = c$ are equations.

The expressions connected by the equality sign are called the **members** of the equation.

34. If the same number is added to each member of an equation or subtracted from each member, the result is an equation equal to the first.

Similarly, if each member of an equation is multiplied or divided by the same number, the result is an equation.

Thus, $3x + 10 = 40$ becomes $3x = 30$ by subtracting 10 from each member.

Also, $3x = 30$ becomes $x = 10$ by dividing each member by 3.

ORAL EXERCISES

1. In $2x + 5 = 25$, subtract 5 from both members. What is the result?

2. In $2x = 30$, divide both members by 2. What is the value of x ?

3. Find x in $2x + 5 = 25$. Also in $3x + 10 = 40$.

Find the value of the letter in each equation.

4. $5x + 5 = 25$.

7. $6 + x = 15$.

5. $8x + 1 = 17$.

8. $20 = 4 + x$.

6. $2y + 12 = 24$.

9. $3z + 2 = 14$.

35. The equation is used in solving problems.

EXAMPLE: Three times a number and 2 more is 14. Find the number.

Solution: Let x = the number.

$$\text{Then, } 3x + 2 = 14. \quad x = \frac{14 - 2}{3} = 4.$$

WRITTEN EXERCISES

1. Five times a number less 6 is 29. What is the number?
2. A house and lot, sold at a gain of 10%, brought \$2,200. What was its former cost?

Plan: (1) Let x be the cost of the property.

(2) Then 110% of x , or $1.10x$, is the selling price.

(3) Therefore, $1.10x = \$2,200$. (4) Therefore, $x = \$(\quad)$.

3. A hardware dealer sold a furnace for \$180 at a gain of 5%. What did the furnace cost him?

4. A merchant sold a damaged carpet for \$42.50 at a loss of 15%. What did the carpet cost him?

5. A collector remitted \$475 after deducting a fee of 5%. How many dollars did he collect?

6. The amount of a certain principal at 4% simple interest for 1 yr. was \$416. What was the principal?

7. A sum of money less 20% is \$960. What is the sum of money?

8. $\frac{3}{4}$ of the height of a staff plus 5 ft. is 25 ft. What is the height of the staff?

9. A commission merchant remitted \$1,900 after deducting a commission of 5%. On how much did he reckon his commission?

10. A man's salary was increased $\frac{1}{4}$ of itself; he then received \$1,800. What was his salary before the increase?

11. George thought of a number, doubled it, added 15, divided by 5 and obtained 5. What was the number?

12. A capital is invested thus: $\frac{1}{4}$ at 5 $\frac{1}{2}$ %, $\frac{1}{4}$ at 6%, and the rest at 4 $\frac{1}{2}$ %; the annual income is \$605. What is the capital?

Plan: (1) Let x be the capital in dollars.

(2) Then, the parts invested are $\frac{1}{4}x$, $\frac{1}{4}x$, $\frac{1}{4}x$.

(3) $.055 \times \frac{1}{4}x + .06 \times \frac{1}{4}x + .045 \times \frac{1}{4}x = \$ (\quad)$, the income.

(4) Therefore, $.0275x + .02x + .0075x = (\quad) x = \605 .

(5) Therefore, $x = \$ (\quad)$.

13. By investing $\frac{1}{4}$ of his capital at 3% and the rest at 6%, Mr. White received \$10 a year more than by investing the whole at 5%. What was his capital?

14. A man sold some of his land at \$400 an acre. He invested the proceeds at 5% and found his daily (365 da. = 1 yr.) income from the investment to be \$1.50. How many acres did he sell?

15. The price of wheat was 78 $\frac{1}{2}$ cts. on Wednesday. It rose a certain amount on Thursday and half as much on Friday, when the price was 80 cts. What was the price on Thursday?

16. A merchant bought 2 canisters of coffee, 50 lb. each, for \$23.25. One kind cost 4 $\frac{1}{2}$ cts. a pound more than the other. Find the price per pound of each.

17. Indicate by use of parentheses, that if twice the cost (c) of a lot be diminished by \$100 and the remainder multiplied by 3, the result will be the cost (h) of the house.

18. Goods are sold for \$1,620 at a loss of 10%. What did they cost?

REVIEW

Regarding $2y$ as one of two factors, state the other factor in each of the following expressions :

1. $4ay$. 2. $20y$. 3. $2y^2$. 4. $30xy$. 5. $6ay^3$.

Perform the operations indicated :

6. $85a + 15a \div 5a$. 9. $8a + 12b \div 4$.
 7. $6z \times (8z + 3z)$. 10. $6y \times 8y + 3y$.
 8. $(85x + 15x) \div 5x$. 11. $(8a + 12b) \div 4$.

Add :

12. $4a + 2$ 13. $2x + 6y$ 14. $a + 5b$ 15. $1 + a$
 $\underline{7a + 9}$ $\underline{3x + 12y}$ $\underline{a + 12b}$ $\underline{2 + 5a}$

Subtract :

16. $12m + 16$ 17. $1 + 12q$ 18. $6x + 27q$ 19. $a + 9b + 5c$
 $\underline{3m + 1}$ $\underline{1 + 4q}$ $\underline{5x + 18q}$ $\underline{a + 2b}$

Indicate by use of exponents :

20. The area of a square $5a$ ft. on a side.
 21. The number of cubic feet in a cube $3x$ ft. on an edge.
 22. The price of 16 lb. of fine wire at $16a$ cts. an ounce.
 23. The number of inches in $36x$ yd.

Multiply :

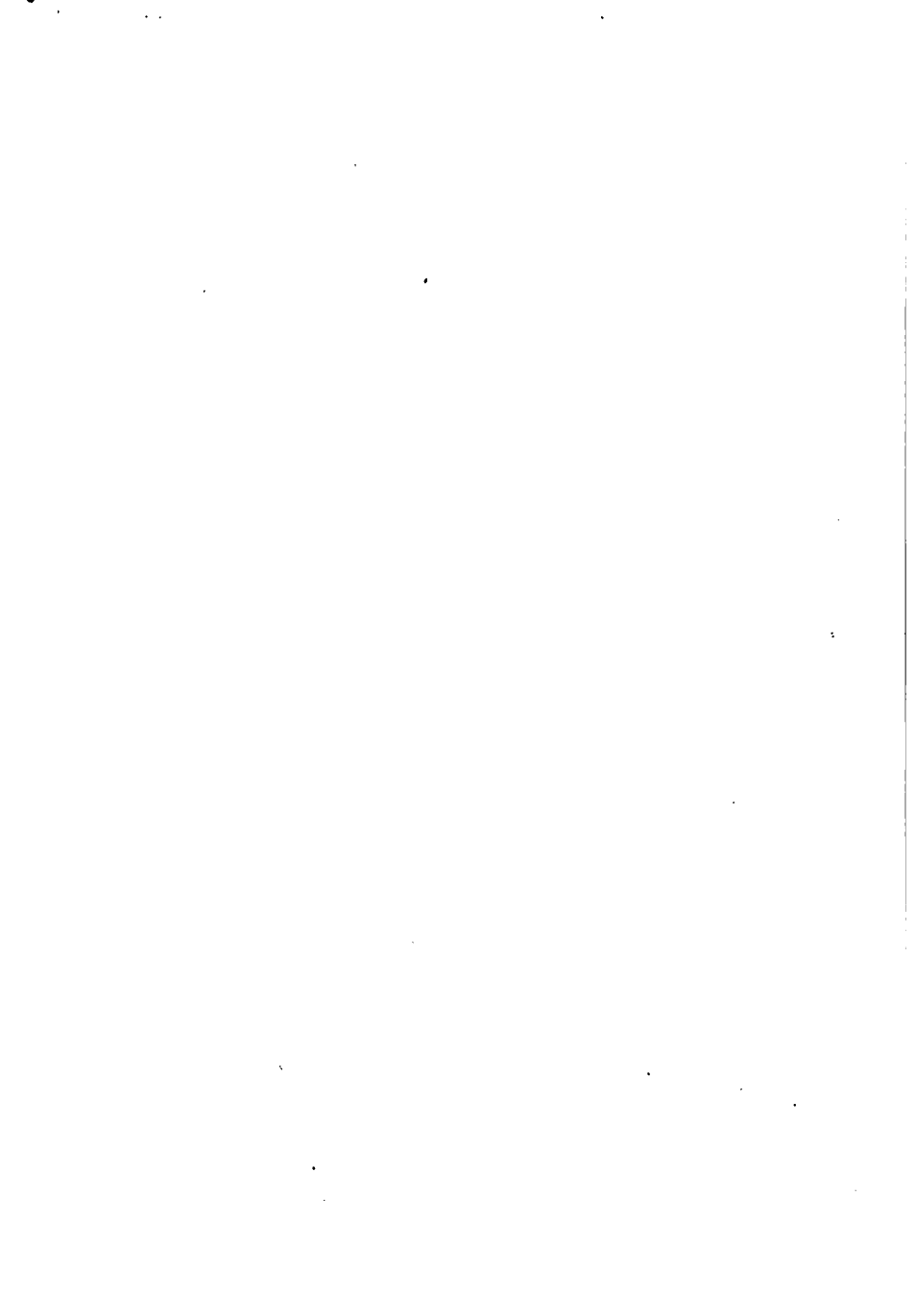
24. $2x + 1$ 25. $5a + b$ 26. $a + m$ 27. $2a - c$
 $\underline{3x + 4}$ $\underline{6a + 3b}$ $\underline{a + m}$ $\underline{a - c}$

Find the square root by factoring :

28. $3136x^2$. 29. $28224m^2$. 30. $9216a^2$. 31. a^4x^8 .

Solve :

32. $2x + 8 = 20$. 34. $x + 17 = 19$. 36. $x + 9 = 29$.
 33. $3x + 3 = 18$. 35. $4x + 1 = 17$. 37. $5x + 6 = 31$.



TABLES OF DENOMINATE NUMBERS

Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
16.5 feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
1,760 yards	= 1 mile
5,280 feet	= 1 mile
6 feet	= 1 fathom

Measure of Time

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days or	
12 months (mo.)	= 1 year (yr.)
10 years	= 1 decade
10 decades	= 1 century

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
231 cu. in.	= 1 gal.
31½ gal.	= 1 bbl.

Table of Counting

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross (gro.)
12 gross	= 1 great gross (gt. gro.)
24 sheets of paper	= 1 quire
20 quires, or 480 sheets,	= 1 ream

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
32 quarts	= 1 bu.
2,150.4 cu. in.	= 1 bu.

Avoirdupois Weight

7,000 grains (gr.)	= 1 pound (lb.)
16 ounces (oz.)	= 1 pound
100 pounds	= 1 hundredweight
2,000 pounds	= 1 ton (T.)
2,240 pounds	= 1 gross ton

United States Money

10 cents (¢)	= 1 dime
10 dimes	= 1 dollar
100 cents	= 1 dollar (\$)
10 dollars	= 1 eagle
20 dollars	= 1 double eagle

Angular Measure

60 seconds (")	= 1 minute (')
60'	= 1 degree (°)
90°	= 1 right angle (rt. ∠)
180°	= 1 straight angle (st. ∠)
360°	= 4 right angles

Surveyors' Measures of Length

7.92 inches	= 1 link (li.)
100 links	= 1 chain (ch.)
80 chains	= 1 mile

The public lands of the United States are surveyed by the chain. Surveyors and engineers now generally use a steel tape graduated in feet and inches or in feet and tenths of a foot.

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet (sq. ft.)	= 1 square yard (sq. yd.)
30 $\frac{1}{2}$ sq. yd.	= 1 square rod (sq. rd.)
160 sq. rd.	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
1 sq. mi.	= a section
36 sq. mi.	= a township
100 sq. ft.	= a square (of roof, etc.)

Cubic Measure

1,728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.)	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord
1 cubic yard	= 1 load (of earth, etc.)
24 $\frac{1}{2}$ cubic feet	= 1 perch (stone)

Apothecaries' Weight

20 grains (gr.)	= 1 scruple (sc. or \mathfrak{D})
8 scruples	= 1 dram (dr. or \mathfrak{ss})
8 drams	= 1 ounce (oz. or \mathfrak{z})
12 ounces	= 1 pound (lb.)
5,760 grains	= 1 pound

The table of apothecaries' weight is used in selling drugs at retail; but the metric system is taking its place with pharmacists.

Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt. or dwt.)
20 pennyweights	= 1 troy ounce
12 troy ounces	= 1 troy pound
480 grains	= 1 troy ounce
5,760 grains	= 1 troy pound

Troy weight is used in weighing precious metals.

For the metric system and moneys of foreign countries see the Supplement, pp. 203-220.

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ANSWERS TO

THE APPLETON ARITHMETICS

THIRD BOOK

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|----------------|--|--------------------|
| 1. 2,088,017. | 4. 875,406.125. | 17. 1,000,000,000. |
| 2. 5,106,200. | 5. 6,805,456,279. | |
| 3. 17,800,625. | 16. 200,000,000 ; 400,000,000 ; 790,000,000. | |

Page 5

- | | | | |
|-------------------------|-------------|-------------------|-------------------|
| 1. 1,987.705 ; 55.3034. | 2. 346,242. | 3. \$ 735,573.07. | 4. \$ 226,537.54. |
|-------------------------|-------------|-------------------|-------------------|

Page 6

- | | | | |
|-------------|-------------|------------|-------------|
| 1. 185,404. | 3. 256,982. | 5. 80,266. | 7. 757,689. |
| 2. 19,000. | 4. 7,203. | 6. 11,864. | 8. 5,146. |

Page 7

- | | | | |
|--------------|----------------|----------------|----------------------|
| 9. 512,849. | 12. 114.74. | 14. 179.53. | 16. 15.120. |
| 10. 158.4. | 13. 2,227,259. | 15. 1,749.126. | 17. \$ 8,684,219.40. |
| 11. 479.653. | | | |

Page 8

- | | | | |
|-----------------------------------|-------------------------------------|--------------|----------------|
| 1. 5,424. | 4. 7,854.37. | 7. 4,328.74. | 10. 999.01. |
| 2. 60.34. | 5. 79.905. | 8. 400.92. | 11. 1,990.18. |
| 3. 6,099. | 6. 666.22. | 9. 117.38. | 12. 1,011.118. |
| 13. \$ 19,754,824 ; \$ 4,372,629. | 14. Liberty National, \$ 1,818,600. | | |

Page 9

15. N. Y. Produce Exchange, \$ 1,283,100 ; New Amsterdam National, \$ 498,200 ; State Bank, \$ 889,000 ; Fourteenth Street Bank, \$ 316,400.
16. \$ 10,393,900 ; \$ 4,354,500 ; \$ 3,769,200 ; \$ 12,067,000 ; \$ 6,049,000.
17. 22,489 ; 40,608.

Page 10

- | | | | |
|--------------|---------------|--------------|--------------|
| 1. \$ 4,815. | 2. \$ 11,495. | 3. \$ 2,892. | 4. \$ 2,932. |
|--------------|---------------|--------------|--------------|

Page 11

5. \$75; \$87.50; \$219; \$861.30; \$4,410; \$1,095; \$328.90; \$956.25;
 \$1,097.60; \$1,584.74; \$11,475; \$40,650.
6. 541.80. 9. 500.04. 12. 11.2518. 15. 75.6851. 18. 2,280.
7. 21,760. 10. 19,680.3 13. 5,742.87. 16. 1,688.4. 19. 11.286.
8. 437.03. 11. 62,264. 14. 36,169.6. 17. 25.375. 20. 2,496.9.

Page 13

1. 207; 486; 684; 873. 11. \$209.58.
2. \$4.23. 12. 1,188; 2,376; 18,414; 9,306.
3. \$34.29. 13. \$13.86.
4. 1,386; 2,673; 5,841; 8,316. 14. 894; 4,768; 11,622; 16,986.
5. \$51.48. 15. \$47.68.
6. \$15.84. 16. \$33.83.
7. 3,996; 50,949; 83,916; 25,974; 717,282. 17. 10,374; 20,748; 1,197; 6,783.
8. 686; 1,470; 3,528; 4,802; 7,056. 18. \$167.52.
9. \$1,176.
10. 597; 1,791; 10,348; 13,731; 16,119; 9,154.

Page 15

1. \$108. 4. \$8.80. 6. \$37.33. 8. \$56.
2. \$49. 5. \$6.75. 7. \$1.87. 9. \$21.88.
3. \$48.

Page 17

1. 72; 320; 9,000.
2. 5^2 ; 2×5^2 ; 3×5^2 ; 2×3^2 ; 3^3 ; $3^2 \times 7$; $2^3 \times 3^2$; $2 \times 3^2 \times 5$.
3. 6×10^2 . 4. 3×10^2 ; 9×10^2 ; 10^3 ; 5×10^3 .
5. 3×10^5 ; 9×10^5 ; 95×10^4 ; 895×10^3 ; 10^6 ; 9×10^6 ; 129×10^6 ; 1236×10^6 .
7. 1492×10^6 ; 1206×10^6 ; 79×10^7 ; 515×10^6 ; 107×10^6 .

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1. 45. 4. 41. 7. 230. 10. $204\frac{1}{4}$. 13. .83. 16. $977\frac{1}{2}$.
2. .55. 5. $201.05\frac{1}{4}$. 8. 77. 11. $264\frac{1}{2}$. 14. 208. 17. 7.47.
3. $118\frac{7}{11}$. 6. 2,001. 9. 3,002. 12. 5. 15. 1,331. 18. $20.05\frac{3}{4}$.

Page 19

19. \$2.97. 21. \$.98. 23. \$1.79. 25. \$3.49. 27. \$.08. 29. \$.08.
20. \$1.25. 22. \$.36. 24. \$8.48. 26. \$1.14. 28. \$5.82. 30. \$1.11.
31. \$3.36. 32. \$9.75. 33. \$.82. 34. \$1.12. 35. \$1,111,111.11. 36. \$625,000.
37. \$2,112,676.06. 38. \$655,737.70. 39. \$146,341.46. 40. \$13.
41. 1840 population = 17,084,766; 1850 population = 23,181,895; 1860 population = 31,445,999; 1870 population = 38,558,713; 1880 population = 50,139,029; 1890 population = 62,647,695; 1900 population = 76,319,437.

Page 20

1870 value = \$866.66; 1880 value = \$736.84; 1890 value = \$840; 1900 value = \$1,058.82.

Page 21

1. 292. 2. 1. 3. 69. 4. 188. 5. 189. 6. 1,924.

Page 22

1. 760. 4. 7,584.8. 7. 50.48. 10. 69.6. 13. 52. 16. 19,552.
2. 4,208. 5. 6,784. 8. 42,920. 11. 4,320. 14. 198.
3. 18.32. 6. 789.6. 9. 264. 12. 824.32. 15. 351.

Page 23

1. 80 yd. 3. 9. 5. 45. 7. 18. 9. 9. 11. 60. 13. 72.
2. 120. 4. 28. 6. 57. 8. 18. 10. 9. 12. 150 A. 14. 24 cords.

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- | | |
|---|---|
| 1. 2×23 . | 17. 37×1 . |
| 2. $2 \times 2 \times 3 \times 3 \times 3$. | 18. $2 \times 2 \times 3 \times 23$. |
| 3. $5 \times 5 \times 5$. | 19. $5 \times 11 \times 11$. |
| 4. 2×19 . | 20. $2 \times 2 \times 2 \times 5 \times 5 \times 5$. |
| 5. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 5$. | 21. $2 \times 2 \times 5 \times 5 \times 19$. |
| 6. $2 \times 5 \times 13 \times 31$. | 22. 3×271 . |
| 7. 1×211 . | 23. $2 \times 3 \times 5 \times 233$. |
| 8. $3 \times 3 \times 3 \times 313$. | 24. 3×811 . |
| 9. 3×37 . | 25. $3 \times 3 \times 17 \times 61$. |
| 10. $2 \times 2 \times 2 \times 2 \times 3 \times 3$. | 26. $2 \times 2 \times 491$. |
| 11. $5 \times 5 \times 5 \times 5$. | 27. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$. |
| 12. 3×17 . | 28. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$. |
| 13. $2 \times 2 \times 2 \times 3 \times 79$. | 29. $2 \times 2 \times 3 \times 3 \times 53$. |
| 14. $7 \times 3 \times 37$. | 30. $2 \times 2 \times 3 \times 11 \times 11$. |
| 15. $2 \times 2 \times 2 \times 2 \times 7 \times 3 \times 3$. | 31. 13×367 . |
| 16. $2 \times 2 \times 479$. | 32. $7 \times 7 \times 13 \times 11$. |

Page 26

1. 26. 3. 14. 5. 12. 7. 12. 9. 25. 11. 7.
2. 18. 4. 19. 6. 5. 8. 15. 10. 21. 12. 5.

Page 27

1. 168. 4. 2,484. 7. 43,095. 10. 625.
2. 2,016. 5. 270. 8. 1,728. 11. 360.
3. 1,694. 6. 1,815. 9. 564,480. 12. 1,728.

Page 29

2. 3,225,000 bales; 967,500 bales; 1,612,500 bales; 645,000 bales; 50%.
 4. 40; 18; 12. Iron, 40,900,000 T.; petroleum, 18,000,000 T.; salt, 12,000,000 T.

Page 30

5. Corn, 25.2+ bu.; wheat, 522,799,200 bu.; oats, 26,976,000 A.
 6. $289\frac{1}{2}$ ft.; $147\frac{1}{2}$ ft.; $93\frac{1}{2}$ ft.
 7. 2.8+ T.; 3.5+ T.; 1.7+ T.; 1.5+ T.; 2.6+ T.; 2.0+ T.
 8. $12\frac{1}{2}$ in.; 36 in.; $10\frac{1}{2}$ in.; $67\frac{1}{2}$ in.; $7\frac{1}{2}$ in.
 9. 73 in.; 70 in.; 50 in.; 34 in.; 25 in.

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- | | | | |
|---------------|---------------|-----------------|------------|
| 1. 313,258. | 4. 2,811.171. | 7. \$546. | 10. 32¢. |
| 2. 397,060. | 5. \$81. | 8. \$520. | 11. \$700. |
| 3. 3,940,581. | 6. \$77.60. | 9. \$3,725,311. | 12. \$408. |

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- | | | | |
|--|-------------------------|-------------------------|---------------------------------|
| 13. $3^2 \times 2$. | 15. $5^2 \times 10^2$. | 17. 2×5^3 . | 19. 10^5 . |
| 14. $3^2 \times 2^2$. | 16. 3^4 . | 18. $10^2 \times 2^3$. | 20. $3^2 \times 2^2 \times 5$. |
| 21. 2×5^2 ; $2^2 \times 5^2$; $2^3 \times 5^3$; $2^2 \times 5^4$; $2^6 \times 5^3$. | 22. 2,210 T. | 23. \$5,495,000. | |
24. Passenger coach, \$5,300; sleeping car, \$14,300; dining car, \$10,900; freight car, \$500.
 25. \$5,290.20; \$14,263.20; \$10,932.51; \$533.16.
 26. 1,008,533 $\frac{1}{2}$ sq. mi. 27. 645,461,333 $\frac{1}{2}$ A.
 28. 40,945,965 A.; 2,537,516 A.; 2,607,089 A.

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- | | |
|-----------------------|------------------------|
| 29. \$18,514.04. | 32. 36,239,000,000 ft. |
| 30. \$104,806,200. | 33. 325 million. |
| 31. \$453,466,887.20. | |
34. Returns per acre, \$30.50; expenses per acre, \$19.95; net profit, \$10.55; net profit from 25 A., \$263.75.
 35. 305. 36. \$9.33. 37. 2,262 thousand ft.; 12 $\frac{1}{2}$.

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- | | | | |
|------------------------------|-------------------------------|---------------|----------------|
| 38. \$48,562.50. | 42. $17\frac{1}{2}$ da. | 46. 800. | 49. \$243,960. |
| 39. \$53,333 $\frac{1}{2}$. | 43. $77\frac{1}{2}$. | 47. \$35,700. | 50. 625. |
| 40. 69,375 bu. | 44. 15,416 $\frac{1}{2}$ bbl. | 48. 321. | 51. \$200,000. |
| 41. \$41,625. | 45. 3. | | |

Page 37

52. \$196.90. 55. $70\frac{7}{8}$ T.; 56. \$229.41. 59. \$630; \$420.
 53. \$33.30. 78.02; \$14.19. 58. $836\frac{4}{11}$ yr. 60. 16,000 A.
 54. $12\frac{3}{4}$ T.

Page 38

61. 696,960,000. 63. $177\frac{7}{8}$ da. 65. \$3,580,000. 67. 81 million T.
 62. 2,400,000 T. 64. \$1,860,000. 66. 300,000 T. 68. 22,800,000,000 lb.

Page 40

1. 70. 7. $\frac{22}{75}, \frac{68}{75}, \frac{28}{75}$. 13. $\frac{32}{100}, \frac{18}{100}, \frac{2}{100}$.
 2. 63. 8. $\frac{720}{1728}, \frac{80}{1728}, \frac{5}{1728}$. 14. $\frac{4}{7}, \frac{2}{7}, \frac{6}{7}$.
 3. 12. 9. $\frac{7}{11}, \frac{4}{11}, \frac{2}{11}$. 15. $\frac{7}{24}, \frac{2}{24}, \frac{2}{24}$.
 4. $\frac{1}{120}, \frac{2}{120}, \frac{20}{120}$. 10. $\frac{2}{240}, \frac{2}{240}, \frac{2}{240}$. 16. $\frac{520}{1248}, \frac{14}{1248}, \frac{1248}{1248}$.
 5. $\frac{2}{24}, \frac{2}{24}, \frac{2}{24}$. 11. $\frac{24}{140}, \frac{14}{140}, \frac{14}{140}$. 17. $\frac{2}{7}, \frac{14}{7}, \frac{14}{7}$.
 6. $\frac{2}{40}, \frac{2}{40}, \frac{2}{40}$. 12. $\frac{2}{120}, \frac{2}{120}, \frac{120}{120}$. 18. $\frac{2}{240}, \frac{2}{240}, \frac{2}{240}$.

Page 41

1. $3\frac{1}{2}$. 11. $2\frac{1}{4}$. 21. $10\frac{7}{8}$. 31. $\frac{23}{2}$. 41. $\frac{46}{2}$.
 2. $1\frac{1}{2}$. 12. $7\frac{1}{2}$. 22. $17\frac{1}{2}$. 32. $\frac{13}{2}$. 42. $\frac{11}{2}$.
 3. $6\frac{1}{2}$. 13. $6\frac{1}{2}$. 23. $6\frac{1}{2}$. 33. $\frac{20}{2}$. 43. $\frac{20}{2}$.
 4. $3\frac{1}{2}$. 14. $10\frac{1}{2}$. 24. $10\frac{1}{2}$. 34. $\frac{57}{2}$. 44. $\frac{57}{2}$.
 5. $6\frac{1}{2}$. 15. $9\frac{1}{2}$. 25. $4\frac{1}{2}$. 35. $\frac{21}{2}$. 45. $\frac{21}{2}$.
 6. $2\frac{1}{2}$. 16. $10\frac{1}{2}$. 26. $\frac{5}{2}$. 36. $\frac{5}{2}$. 46. $\frac{5}{2}$.
 7. $5\frac{1}{2}$. 17. $2\frac{1}{2}$. 27. $\frac{4}{2}$. 37. $\frac{47}{2}$. 47. $\frac{47}{2}$.
 8. $4\frac{1}{2}$. 18. $12\frac{1}{2}$. 28. $\frac{5}{2}$. 38. $\frac{17}{2}$. 48. $\frac{17}{2}$.
 9. $5\frac{1}{2}$. 19. $18\frac{1}{2}$. 29. $\frac{5}{2}$. 39. $\frac{12}{2}$. 49. $\frac{12}{2}$.
 10. $6\frac{1}{2}$. 20. $17\frac{1}{2}$. 30. $\frac{12}{2}$. 40. $\frac{17}{2}$. 50. $\frac{22}{2}$.
 1. $\frac{3}{4}$. 4. $\frac{1}{4}$. 7. $\frac{7}{4}$. 10. $\frac{1}{4}$. 13. $\frac{6}{4}$. 16. $\frac{13}{4}$. 19. $\frac{13}{4}$.
 2. $\frac{3}{4}$. 5. $\frac{5}{4}$. 8. $\frac{3}{4}$. 11. $\frac{1}{4}$. 14. $\frac{7}{4}$. 17. $\frac{5}{4}$. 20. $\frac{5}{4}$.
 3. $\frac{3}{4}$. 6. $\frac{1}{4}$. 9. $\frac{3}{4}$. 12. $\frac{3}{4}$. 15. $\frac{2}{4}$. 18. $\frac{1}{4}$. 21. $\frac{17}{4}$.

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22. 0. 26. $1\frac{1}{2}$. 30. $1\frac{1}{2}$. 34. $\frac{17}{2}$. 38. $1\frac{1}{2}$.
 23. $1\frac{1}{2}$. 27. $1\frac{1}{2}$. 31. $\frac{3}{2}$. 35. $1\frac{1}{2}$. 39. $\frac{3}{2}$.
 24. $\frac{48}{100}$. 28. $\frac{3}{2}$. 32. $\frac{1}{2}$. 36. $2\frac{1}{2}$. 40. $3\frac{1}{2}$ mi.
 25. $1\frac{1}{2}$. 29. $\frac{1}{2}$. 33. $\frac{1}{2}$. 37. $1\frac{1}{2}$. 41. $\frac{1}{2}$.

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- | | | | | | |
|---------------------------------------|------------------------|-----------------------|--------------------------|------------------------|------------------------|
| 1. Improper fraction; $\frac{4}{3}$. | 2. $28\frac{1}{2}$. | 3. $1\frac{1}{2}$. | 4. $\frac{2}{3}$. | 5. $11\frac{1}{2}$. | 6. $\frac{2}{3}$. |
| 7. $28\frac{1}{2}$. | 14. $3\frac{1}{2}$. | 21. $6\frac{1}{2}$. | 28. $72\frac{1}{2}$. | 35. $23\frac{1}{2}$. | 42. $31\frac{1}{2}$. |
| 8. $163\frac{1}{2}$. | 15. $2\frac{1}{2}$. | 22. $14\frac{1}{2}$. | 29. $91\frac{1}{2}$. | 36. $22\frac{1}{2}$. | 43. $158\frac{1}{2}$. |
| 9. $58\frac{1}{2}$. | 16. $\frac{1}{2}$. | 23. $2\frac{1}{2}$. | 30. $132\frac{1}{2}$. | 37. $167\frac{1}{2}$. | 44. $51\frac{1}{2}$. |
| 10. $97\frac{1}{2}$. | 17. $40\frac{1}{2}$. | 24. $2\frac{1}{2}$. | 31. $6\frac{1}{2}$. | 38. $318\frac{1}{2}$. | 45. 170. |
| 11. $10\frac{1}{2}$. | 18. $77\frac{1}{2}$. | 25. $\frac{2}{3}$. | 32. $2\frac{2}{3}$. | 39. $64\frac{1}{2}$. | 46. $569\frac{1}{2}$. |
| 12. $9\frac{1}{2}$. | 19. $76\frac{1}{2}$. | 26. $\frac{2}{3}$. | 33. $140\frac{1}{2}$. | 40. $744\frac{1}{2}$. | 47. $405\frac{1}{2}$. |
| 13. $3\frac{1}{2}$. | 20. $128\frac{1}{2}$. | 27. $74\frac{1}{2}$. | 34. $1,100\frac{1}{2}$. | 41. $578\frac{1}{2}$. | 48. $36\frac{1}{2}$. |

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- | | | | |
|---|---------------------------|--|-------------------------|
| 49. $38\frac{1}{2}$. | 50. $8\frac{1}{2}$ yd. | 51. $1\frac{1}{2}$ yd. | 52. $202\frac{1}{2}$ T. |
| 53. Tuesday; Monday, $\frac{1}{2}$ higher; Wednesday, $3\frac{1}{2}$ higher; Thursday, $5\frac{1}{2}$ higher; Friday, $3\frac{1}{2}$ higher; Saturday, $2\frac{1}{2}$ higher. | | | |
| 54. $1\frac{1}{2}$ ft. | 56. 495 ft. | 58. $44\frac{1}{2}$ mi. | |
| 55. $10\frac{1}{2}$ ft.; $1\frac{1}{2}$ ft. | 57. $278\frac{1}{2}$ gal. | 59. $24\frac{1}{2}$ mi.; $19\frac{1}{2}$ mi. | |

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- | | | | | |
|---------------------|------------------------|------------------------|---------------------------|-----------------------------------|
| 1. 3. | 9. 4. | 17. $\frac{2}{3}$. | 25. $24\frac{1}{2}$. | 33. 700. |
| 2. $3\frac{1}{2}$. | 10. $\frac{1}{2}$. | 18. $57\frac{1}{2}$. | 26. $42,013\frac{1}{2}$. | 34. $365\frac{1}{2}$. |
| 3. $\frac{1}{2}$. | 11. $\frac{1}{2}$. | 19. $389\frac{1}{2}$. | 27. $86\frac{1}{2}$. | 35. $82\frac{1}{2}$. |
| 4. $\frac{1}{2}$. | 12. $\frac{1}{2}$. | 20. $6\frac{1}{2}$. | 28. 60. | 36. $187\frac{1}{2}$. |
| 5. 36. | 13. $36\frac{1}{2}$. | 21. $2\frac{1}{2}$. | 29. $\frac{1}{2}$. | 37. The first; $391\frac{1}{2}$. |
| 6. $\frac{1}{2}$. | 14. $391\frac{1}{2}$. | 22. $\frac{1}{2}$. | 30. 2,200. | 38. $\frac{1}{2}$ yd. |
| 7. $\frac{1}{2}$. | 15. $68\frac{1}{2}$. | 23. $\frac{1}{2}$. | 31. $\frac{1}{2}$. | 39. \$3.92. |
| 8. $2\frac{1}{2}$. | 16. $175\frac{1}{2}$. | 24. $17\frac{1}{2}$. | 32. $6\frac{1}{2}$. | 40. $84\frac{1}{2}$ cu. yd. |

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- | | | |
|-----------------------------|--------------------------|--|
| 41. 584 T. | 45. 103 sq. ft. | 49. $66\frac{1}{2}$ ft. |
| 42. \$9,932.40. | 46. $2,595\frac{1}{2}$. | 50. 341 ft. |
| 43. $94\frac{1}{2}$ sq. yd. | 47. \$8.87. | 51. $3\frac{1}{2}$ ft.; $2\frac{1}{2}$ ft.; $2\frac{1}{2}$ ft.; $1\frac{1}{2}$ ft. |
| 44. $380\frac{1}{2}$ mi. | 48. $39\frac{1}{2}$ ft. | |

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- | | | |
|---|-----------------------|-------------------|
| 52. $\frac{1}{2}$. | 53. 180 lb.; \$28.35. | 54. \$41,040. |
| 55. Mexico, \$45,360; Costa Rica, \$41,040; Guatemala, \$41,760; Venezuela, \$38,880; Colombia, \$38,880; Brazil, \$26,640. | | |
| 56. 1,460,250,000 lb. | 57. \$135,073,125. | 58. 63,675 lb. |
| | | 59. \$13,365,000. |

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|----------------------|----------------------|----------------------|-----------------------|--------------------------|----------------------|
| 1. $1\frac{1}{2}$. | 10. 2. | 19. 40. | 28. $\frac{1}{2}$. | 37. $1\frac{1}{2}$. | 46. $2\frac{1}{2}$. |
| 2. $18\frac{1}{2}$. | 11. $1\frac{1}{2}$. | 20. $4\frac{1}{2}$. | 29. $\frac{1}{2}$. | 38. $\frac{1}{2}$. | 47. $1\frac{1}{2}$. |
| 3. 225. | 12. $1\frac{1}{2}$. | 21. $\frac{1}{2}$. | 30. 4. | 39. 1. | 48. $3\frac{1}{2}$. |
| 4. 128. | 13. $\frac{1}{2}$. | 22. 9. | 31. $6\frac{1}{2}$. | 40. $2,666\frac{1}{2}$. | 49. $1\frac{1}{2}$. |
| 5. 2. | 14. $1\frac{1}{2}$. | 23. $8\frac{1}{2}$. | 32. $83\frac{1}{2}$. | 41. $1\frac{1}{2}$. | 50. 3. |
| 6. $\frac{1}{2}$. | 15. 2. | 24. 81. | 33. 192. | 42. $\frac{1}{2}$. | 51. $1\frac{1}{2}$. |
| 7. $1\frac{1}{2}$. | 16. 240. | 25. $\frac{1}{2}$. | 34. $68\frac{1}{2}$. | 43. $3\frac{1}{2}$. | 52. $\frac{1}{2}$. |
| 8. 30. | 17. $3\frac{1}{2}$. | 26. $2\frac{1}{2}$. | 35. 3. | 44. $\frac{1}{2}$. | |
| 9. $\frac{1}{2}$. | 18. 28. | 27. $1\frac{1}{2}$. | 36. $1\frac{1}{2}$. | 45. 1000. | |

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|-----------------------|----------------------|----------------------------|----------------------------|-----------------------|
| 53. $5\frac{1}{2}$. | 56. $3\frac{1}{2}$. | 59. $6\frac{1}{2}$. | 62. $21\frac{1}{2}$. | 65. $34\frac{1}{2}$. |
| 54. $34\frac{1}{2}$. | 57. $\frac{1}{2}$. | 60. 186. | 63. 16. | 66. 20. |
| 55. $11\frac{1}{2}$. | 58. $3\frac{1}{2}$. | 61. 83; $2\frac{1}{2}$ ft. | 64. 252; $1\frac{1}{2}$ T. | 67. $27\frac{1}{2}$. |

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|---------------------|--------------------|---------------------|---------------------|----------------------|
| 1. $\frac{1}{2}$. | 3. $\frac{1}{2}$. | 5. $\frac{1}{2}$. | 7. $2\frac{1}{2}$. | 9. $26\frac{1}{2}$. |
| 2. $4\frac{1}{2}$. | 4. $\frac{1}{2}$. | 6. $2\frac{1}{2}$. | 8. $4\frac{1}{2}$. | |

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|-----------------------|------------------------|------------------------|----------------------|-----------------------|
| 1. $3\frac{1}{2}$. | 12. $1\frac{1}{2}$. | 23. 20. | 34. $\frac{1}{2}$. | 45. 96. |
| 2. $6\frac{1}{2}$. | 13. $1\frac{1}{2}$. | 24. $423\frac{1}{2}$. | 35. $\frac{1}{2}$. | 46. $1\frac{1}{2}$. |
| 3. $1\frac{1}{2}$. | 14. $\frac{1}{2}$. | 25. $895\frac{1}{2}$. | 36. $1\frac{1}{2}$. | 47. $\frac{1}{2}$. |
| 4. $1\frac{1}{2}$. | 15. $\frac{1}{2}$. | 26. $1\frac{1}{2}$. | 37. $1\frac{1}{2}$. | 48. 6. |
| 5. $\frac{1}{2}$. | 16. $\frac{1}{2}$. | 27. $\frac{1}{2}$. | 38. $7\frac{1}{2}$. | 49. $2\frac{1}{2}$. |
| 6. 24. | 17. $1\frac{1}{2}$. | 28. $\frac{1}{2}$. | 39. $1\frac{1}{2}$. | 50. $4\frac{1}{2}$. |
| 7. $17\frac{1}{2}$. | 18. $47\frac{1}{2}$. | 29. $107\frac{1}{2}$. | 40. $\frac{1}{2}$. | 51. $3\frac{1}{2}$. |
| 8. $4\frac{1}{2}$. | 19. $112\frac{1}{2}$. | 30. $309\frac{1}{2}$. | 41. $\frac{1}{2}$. | 52. $1\frac{1}{2}$. |
| 9. $9\frac{1}{2}$. | 20. $\frac{1}{2}$. | 31. 76. | 42. $\frac{1}{2}$. | 53. $5\frac{1}{2}$. |
| 10. $6\frac{1}{2}$. | 21. $\frac{1}{2}$. | 32. 240. | 43. $1\frac{1}{2}$. | 54. $\frac{1}{2}$. |
| 11. $12\frac{1}{2}$. | 22. $\frac{1}{2}$. | 33. $1\frac{1}{2}$. | 44. $4\frac{1}{2}$. | 55. $55\frac{1}{2}$. |

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|---------------------------------|--------------------------|---|
| 56. $53\frac{1}{2}$ sq. ft. | 59. \$18.02. | 62. \$421.10. |
| 57. \$354.60. | 60. \$6.39. | 63. \$1. |
| 58. $73\frac{1}{2}$; \$458.91. | 61. \$525; \$360.94. | 64. \$40.91. |
| 65. \$9.59. | 66. 116.12 gr.; 12.9 gr. | 67. 19.29 gr. nickel; 57.87 gr. copper. |

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68. Dining room, $49\frac{1}{2}$ ft.; living room, $60\frac{1}{2}$ ft.; parlor, $61\frac{1}{2}$ ft.; library, 49 ft.; kitchen, $44\frac{1}{2}$ ft.
69. $12\frac{1}{2}$ ft. 74. \$42,857.14.
70. 44 ft. 75. Length, 3 ft. $6\frac{1}{8}$ in.; width, $8\frac{1}{2}$ in.
72. 371.25 gr. 76. Length, $18\frac{1}{8}$ in.; width, $15\frac{1}{8}$ in.
73. 420 sq. in.; $11\frac{1}{2}$ ft.; \$17.14.

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78. $67\frac{1}{2}$ ft.; $4\frac{1}{2}$ ft. 80. 12 hr.; $1\frac{1}{2}$ da. 82. $25\frac{1}{2}$ ft.; $8\frac{1}{2}$ yd.
79. 19 ft. 1 in.; 15 ft. 81. \$.56. 83. $42\frac{1}{2}$ ft.

Page 60

1. 163,208 oz. 4. 9,660 cu. in. 7. 504,390 sec. 10. 2,132 in.
2. 143 pt. 5. 2,346,629 cu. in. 8. 145,245". 11. 2,418 sheets.
3. 982 units. 6. 7,922 ft. 9. 415 qt. 12. 14,152 sq. in.

Page 61

1. 15 gal. 2 qt. 1 pt. 8. 1 hr. 34 min. 50 sec.
2. 2,065 gal. 3 qt. 9. 2 lb. 4 oz. 10 gr.
3. 10 bu. 7 qt. 10. 21 rd. 13 ft. 11 in.
4. 109 gal. 1 qt. 1 pt. 11. 1 mi. 5,098 ft.
5. 15 bu. 1 pk. 5 qt. 12. 4 sq. yd. 4 sq. ft. 130 sq. in.
6. 2 T. 956 lb. 13. 9 cu. ft. 211 cu. in.
7. 3 da. 7 hr. 23 min.

Page 62

1. 1 hr. 18 min.; 2 hr. 42 min.; 4 hr. 54 min.; 12 hr. 36 min.; 30 min.
2. 22 hr. To add without reducing is shorter.
3. 7 yd. 2 ft. 4.8 in.; 5 yd. 9 in. 9. 2.875 gal.; 5.375 gal.
4. 272 sq. rd. 10. 6.5 sq. ft.; $15.86\frac{2}{3}$ sq. ft.
5. 83.25 sq. ft. 11. 1.90+ sq. yd.
6. $\frac{5}{8}$ yd. 12. 4.336+ cu. yd.
7. 198 oz. 13. 6.75 ft.; 2.8 ft.; 7.65 ft.; 8.92 ft.
8. $5.66\frac{2}{3}$ ft.; $9.41\frac{2}{3}$ ft.
14. $26\frac{2}{3}$ ft.; 26.12 ft. Adding with decimals is shorter.

Page 63

1. 23 gal. 3 qt. 2. 16 pk. 6 qt. 3. 45 mi. 202 rd. 4. 33 yd. 1 ft. 8 in.

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|--|--------------------------------------|
| 5. 31 da. 7 hr. 1 min. | 14. 2 cu. yd. 20 cu. ft. 828 cu. in. |
| 6. 69 bu. 3 pk. 6 qt. | 15. 25 bu. 1 pk. 7 qt. |
| 7. 29 cu. yd. 23 cu. ft. 1,110 cu. in. | 16. 14 sq. yd. 7 sq. ft. 69 sq. in. |
| 8. 101 A. 20 sq. rd. 13 sq. yd. | 17. 38 yd. 1 ft. 3 in. |
| 9. 33 wk. 1 da. 18 hr. | 18. 5 hr. 48 min. 26 sec. |
| 10. 65 sq. yd. 5 sq. ft. 21 sq. in. | 19. 17 min. 45 sec. |
| 11. 10 wk. 5 da. 16 hr. | 20. 4 gal. 2 qt. 1 pt. |
| 12. 9 mi. 260 rd. 5 ft. | 21. 2 min. 32½ sec. |
| 13. 8 yd. 4 in. | |

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|--|---|
| 1. 24 gal. 1 qt. 1 pt. | 6. 11.75 cu. yd. 11 cu. ft. 39 cu. in. |
| 2. 59 pk. 3.5 qt. 1.5 pt. | 7. 4 da. 4 hr. 50 min. 40 sec. |
| 3. 35 bu. 1 pk. 7 qt. | 8. 3 sq. mi. 564 A. 114 sq. rd. 7.5 sq. yd. |
| 4. 162 mi. 19 rd. 10.5 ft. | 9. 23 yd. 1 ft. 4 in.; 70½ ft. |
| 5. 201 sq. rd. 128.75 sq. ft. 96 sq. in. | 10. 376½ ft. |

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|--|-------------------------------|
| 11. 4.75 bu.; 2.25 bu.; 6.12½ bu.; 8.37½ bu. | |
| 12. 47 bu. 2 pk.; 22 bu. 2 pk.; 61 bu. 1 pk.; 83 bu. 3 pk. | |
| 13. 47.5 bu.; 22.5 bu.; 61.25 bu.; 83.75 bu. Multiplying the decimal equivalents is shorter. | |
| 14. 4.875 gal.; 24.375 gal. | 15. 10.128+ mi.; 162.048+ mi. |

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|---------------------------------------|--------------------------------------|
| 1. 41 lb. 8 oz.; 920 lb.; 1 bu. 4 qt. | 4. 245 rd. 9 in. |
| 2. 2 bu. 2 pk. ¾ qt. | 5. 7 cu. yd. 22 cu. ft.; 672 cu. in. |
| 3. 7 gal. 1 qt. | 6. 64 sq. ft. 20 sq. in. |
| 7. 407½ lb. | 11. 104½. |
| 8. 1 hr. 22 min. 1 sec. | 12. 2 yd. 1⅞ in. |
| 9. 5 hr. 50 min. | 13. 8¾. |
| 10. 1 bu. 2 pk. 1⅞ pt. | 14. 5 rd. 3 ft. 7 in. |
| 19. 12½. | 20. 100½. |
| | 21. 5½. |
| | 22. 7. |
| | 15. 7 yd. 1 ft. 2 in. |
| | 16. 45½. |
| | 17. 625.09½ bu. |
| | 18. 80. |

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|---------------------------|------------------|----------------------|
| 1. 1,728 cu. in.; 7¾ gal. | 3. 2,692¾ gal. | 5. 114 bbl., nearly. |
| 2. 53½ cu. ft. | 4. ¾ cu. ft. | |
| 1. 10,752.1 cu. in. | 3. 4.99+ bu. | 5. 774.19+ bu. |
| 2. 1,612,815 cu. in. | 4. 1.24+ cu. ft. | 6. 128,000 bu. |

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7. 67.20 cu. in. ; 57.75 cu. in. The dry quart is 9.45 cu. in. larger.

1. $\frac{11}{17}$.

3. 9.504 gr. ; .0217+.

5. $\frac{111}{171}$.

2. $\frac{144}{171}$.

4. 437 $\frac{1}{2}$ gr.

Page 71

1. $19^{\circ} 44' 13.5''$; $171^{\circ} 15' 31.5''$; $11^{\circ} 3' 28.5''$; $126^{\circ} 18' 46.5''$.

Page 72

1. 43 min. 22 $\frac{1}{2}$ sec.

4. 57 min. 23 sec.

7. 1 hr. 40 min. 49 $\frac{1}{2}$ sec.

2. 21 min. 13 sec.

5. 6 hr. 8 $\frac{1}{2}$ sec.

8. 1 hr. 8 min. 1 $\frac{1}{5}$ sec.

3. 32 min. 39 $\frac{1}{2}$ sec.

6. 50 min. $\frac{1}{2}$ sec.

9. 12 hr. 2 min. 40 sec., or 11 hr. 57 min. 20 sec. in the opposite direction.

10. $125^{\circ} 3' 45''$.

12. $88^{\circ} 44'$

14. $181^{\circ} 2' 30''$.

11. $91^{\circ} 10'$.

13. $150^{\circ} 11' 15''$.

15. $350^{\circ} 4'$, or $9^{\circ} 56'$.

Page 74

1. 133 pt.

9. 16.875 gal.

16. 23,850'' ; 8,400''.

2. 31,018 in.

10. $5.95\frac{1}{2}$ bu.

17. 12 wk. 3 da. 13 hr.

3. 938 in.

11. 17.5 ft.

18. 44 gal. 1 qt. 1 pt.

4. 6,868 sq. in.

12. 666 sq. yd. 6 sq. ft.

19. $41^{\circ} 8' 24''$.

5. 9,765 cu. in.

13. $40^{\circ} 30' 59''$; $84^{\circ} 55' 43''$.

20. 31 bu. 2 pk. 4 qt.

6. 1 T. 507 lb. 13 oz.

14. $60''$; $150''$; $345''$.

21. 91 sq. yd. 2 sq. ft.

7. 1 mi. 5 in.

15. $90'$; $285'$.

22. $47^{\circ} 3' 13''$.

8. 160 cu. yd. 7 cu. ft.

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23. 2 gal. 2 qt.

33. 20 bu. 1 qt. $\frac{1}{4}$ pt.

24. $28^{\circ} 44' 5''$.

34. $3^{\circ} 4' 48\frac{1}{2}''$.

25. 1 mi. 319 rd. 14.5 ft.

35. 159.58 cu. in.

26. $40^{\circ} 32' 35''$.

36. $1\frac{1}{4}\frac{1}{4}$ lb. troy.

27. 64 T.

37. 1,478.4 cu. in.

28. $131^{\circ} 24'$.

38. 23 lb. 12 oz.

29. 44 cu. yd. 12.5 cu. ft.

39. 8 $\frac{1}{2}$ qt.

30. $132^{\circ} 43' 12''$.

40. \$15.17.

31. 24 yd. 2 ft. 2 $\frac{1}{4}$ in.

41. $76^{\circ} 18' 40''$.

32. $1^{\circ} 24' 1''$.

42. 5 hr. 5 min. 14.7 sec.

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43. 358 lb. 8 oz.

46. 194 ft. 10 in.

49. 3 gal. $\frac{1}{2}$ pt.

44. 10 $\frac{1}{2}$ lb.

47. 8 gal. 1 pt.

50. 122 A. 80 sq. rd.

45. 8 ft.

48. 2 wk. 4 da. 4 hr.

51. \$1,408.88.

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52. 1 P.M.; 10 A.M.; 11 A.M.; 7 P.M.; 6 P.M.; 1 A.M. of the following day.
 53. 9 min. 33½ sec. past noon; noon; 9 min. 33½ sec.
 54. 960 rd. 56. 30½ hr. 58. \$224.50.
 55. $(16\frac{1}{2})^2$; 43,560 sq. ft. 57. \$87.50. 59. \$123.68.

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1. 32¢. 2. \$3.25. 3. 7,000. 4. 2,072.

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1. 60 yd. 2. 4 hr. 32 min. 3. \$52. 4. 480 pages.

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5. $\frac{980}{8}$ yd.; $\frac{980}{16 \times 8}$ yd.; $\frac{20 \times 980}{16 \times 8}$ yd.; $\frac{28 \times 20 \times 980}{16 \times 8}$ yd. = 4,287½ yd.

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1. 9. 2. 10. 3. 8. 4. 62.

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5. 5. 7. 10. 9. 40 mi. 11. 945 mi. 13. 6.
 6. 23. 8. \$1,250. 10. 285 mi. 12. 297 mi. 14. 2.
 15. John, 11 yr.; James, 15 yr. 16. 5½ ft.

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17. 2½ ft.; 1½ ft. 18. Tower, 80 ft.; staff, 20 ft. 19. House, \$3,000; lot, \$500.
 20. Part out of water, 27 ft.; part under water, 18 ft.
 21. 139½ lb. 22. 63 lb.
 23. 1⅞ lb. starchy foods. ⅓ lb. fats. ⅓ lb. albuminous foods.
 24. 5 bu. 25. 656. 26. 73 lb.

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1. 175 ml. 2. \$153.60. 3. 5,400.

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1. A, \$272.½. B, \$727.½. C, \$1000.
 2. A, \$200; B, \$600; C, \$300; D, \$1,300.
 3. 1st, \$20; 2d, \$40; 3d, \$80.
 4. 362.88 cu. in. of oxygen; 1,365.12 cu. in. of nitrogen.

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1. 60 lb., 70 lb. 4. \$900. 7. \$12,600; \$2,400.
 2. 9 yr. 5. \$3,125; \$1,875. 8. 960 A.; 15½%.
 3. \$1000. 6. \$7,951.50; \$9,718.50. 9. 5 yr.

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|---|---------------------|------------------------|
| 10. \$1.60. | 13. 115,028 sq. mi. | 16. 4,277,170 sq. mi. |
| 11. 590,884 sq. mi. | 14. 6,736 sq. mi. | 17. 11,286,344 sq. mi. |
| 12. 3,435 sq. mi. | 15. 209,358 sq. mi. | 18. 208,830 sq. mi. |
| 19. 1st, \$2,400; 2d, \$3,600; 3d, \$6,000. | | |

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|-------------|------------|-----------|-------------|--------------|--------------|
| 1. 80.475. | 4. 28.152. | 7. 59.67. | 10. 150.41. | 13. 9.666. | 16. 24.723. |
| 2. 45.7875. | 5. 14.11. | 8. 148.8. | 11. 1.207. | 14. .00153. | 17. 756.6. |
| 3. 258. | 6. 2.375. | 9. 70. | 12. 2.66. | 15. 1.59185. | 18. 1.76625. |

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|--|---------------|
| 19. \$42; \$661.50; \$4,100.733. | |
| 20. \$492.081; \$1,485.75; \$808.21; \$950.48. | 21. 186.08 A. |

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|---|--|---|
| 1. 57; 19. | 2. 20; 5. | 3. 80 sq. ft.; 300 sq. ft.; 100 sq. ft. |
| 4. 30 min.; 1,600 cu. ft.; 4,000 cu. ft.; 8,000 cu. ft. | | |
| 5. 115; 172; \$20.04; \$21.16; 316. | | |
| 6. 121; 230; \$60.22; \$120.30; \$101.21. | | |
| 7. 32; \$41.02; \$42.10; \$21.06; 18. | | |
| 8. 60; 135; \$150.30; \$375.15; \$213.15. | | |
| 9. 600; \$570.30; \$750.60; \$903.45; 675. | | |
| 10. 140; 315; 350.70; 875.35; 497.35. | | |
| 11. 242; 460; \$120.44; \$240.60; \$202.42. | | |
| 12. 729; 418.5; 776.7; \$90.18; \$904.50. | | |
| 13. 19.63; 180.5; 1.725; \$4.09; 176.2. | 14. \$30.663; \$4.60; 4.481; 6771; 1531. | |

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|--|---|
| 1. $4\frac{1}{2}\%$; 4% ; $3\frac{1}{2}\%$. | 5. $84\frac{1}{3}\%$; $15\frac{1}{3}\%$. |
| 2. $4\frac{1}{11}\%$; $16\frac{2}{3}\%$; $12\frac{1}{2}\%$; 900% . | 6. 2% ; 98% . |
| 3. $10\frac{7}{8}\%$; $1\frac{1}{8}\%$; $3\frac{1}{8}\%$. | 7. $2,083\frac{1}{3}$; $2\frac{1}{11}$; $2\frac{1}{12}$; $2\frac{1}{12}\%$. |
| 4. $90\frac{1}{11}\%$; $9\frac{1}{11}\%$. | 8. 85% . |

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|--------------------------------|----------|------------|
| 1. \$500. | 4. 2621. | 7. 1,0661. |
| 2. 1000 sq. ft.; 1,775 sq. ft. | 5. 600. | 8. 1,728. |
| 3. 170. | 6. 560. | 9. 310. |

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|--|--------------------------|--|------------|-------------|
| 1. 1; 100%. | 4. $\frac{1}{2}$; 20%. | 7. 12¢; $16\frac{2}{3}\%$. | 10. \$70. | 13. \$300. |
| 2. $\frac{1}{10}$; 30%. | 5. 18¢; $\frac{1}{2}$. | 8. 9¢; $33\frac{1}{3}\%$. | 11. \$500. | 14. \$200. |
| 3. $\frac{1}{3}$; $11\frac{1}{3}\%$. | 6. 24¢; $\frac{1}{12}$. | 9. $17\frac{1}{2}\%$; $14\frac{2}{3}\%$. | 12. \$200. | 15. \$1000. |

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|--------------|-----------------------|----------------|
| 16. 25% | 19. \$5,285; 19% | 22. \$300; 31% |
| 17. 40% | 20. \$4,738.50; 27.1% | 23. 16½% |
| 18. \$5,850. | 21. \$150; 2½% | 24. 106½% |

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|--------------|-------------|--------------|-----------------|
| 1. \$109.38. | 4. \$52. | 7. \$2.81. | 10. \$1.23. |
| 2. \$32.81. | 5. \$46. | 8. \$15.99. | 11. \$18.75; 5% |
| 3. \$26.13. | 6. \$22.80. | 9. \$12.958. | 12. \$425. |

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|--------------|--------------|--------------|--------------|-------------|
| 13. \$5,850. | 15. \$17.50. | 17. \$24.44. | 19. \$45.50. | 21. \$413. |
| 14. \$110. | 16. \$8,000. | 18. \$5.25. | 20. \$379. | 22. \$9.83. |

1. 42.9+%
2. $32\frac{1}{2}\%$; 50%; $48\frac{7}{10}\%$.
3. The prices are identical, \$166.32; the discount in each case equaling 49 $\frac{1}{2}\%$.
4. \$376.32.
5. (1) \$34.56. (2) \$9.22. (3) \$69.84. (4) \$169.75. (5) \$121.25.
(6) \$33.10. (7) \$11.06. (8) \$93.36. (9) \$73.72.

- 1. \$31,600. 2. 31 $\frac{1}{8}$ %; 53 $\frac{8}{8}$ %. 4. \$1,580,000.**

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|------------------|--|---------------|------------|
| 1. 232.74. | 3. 62.80. | 5. 3,711. | 7. 27,048. |
| 2. 123. | 4. 5,245.8. | 6. 459.27. | 8. 7,895. |
| 9. 15,264.40. | 10. 44 $\frac{1}{2}$ % ; 27 $\frac{1}{2}$ % ; 16 $\frac{3}{4}$ % ; 11 $\frac{1}{2}$ %. | | |
| 11. \$ 1,002.50. | 13. 1,775,500. | 15. 5% ; .7%. | |
| 12. \$ 600. | 14. 23.7%. | 16. 22.3%. | |

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|---------------------------|-------------------------|----------------|------------|----------|
| 17. 2 %. | 19. 46 $\frac{1}{2}$ %. | 21. \$ 15,000. | 23. .3 %. | 25. 24 % |
| 18. 412 $\frac{1}{4}$ gr. | 20. 45 lb. | 22. \$ 45. | 24. 9.8 %. | |

- 28.** Decrease of 9.1 %.
- 29.** Decrease of 12.5 %; decrease of 5.4 %; decrease of 3.8 %.
- 31.** \$2,356.06. **33.** \$377.34. **35.** \$2,000.
- 32.** \$395.88. **34.** \$3,010. **36.** \$1,500.

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37. 54+%. 38. 93+%. 39. 29,069. 40. 174,400. 41. 22,464 sq. mi. 42. $\frac{1}{2}\%$
 43. 1st, \$475; 2d, \$250; 3d, \$50; 4th, \$25; 5th, \$5.
 44. 1st, \$2,375; 2d, \$2,500; 3d, \$1,250; 4th, \$1,250; 5th, \$125.
 45. 1st, \$479.75; 2d, \$252.50; 3d, \$50.50; 4th, \$25.25; 5th, \$5.05.
 46. \$1,600; \$14.

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|--------------|------------|-------------|-------------|
| 1. \$70.88. | 4. \$5.25. | 7. \$256. | 10. \$1.50. |
| 2. \$91. | 5. \$4.52. | 8. \$78.75. | 11. \$4.66. |
| 3. \$221.67. | 6. \$6. | 9. \$185. | 12. \$5.04. |
13. \$1,554,588.33.
 14. Chicago, \$180,000; Philadelphia, \$193,333.33; St. Louis, \$80,000; Boston, \$293,333.33.

Page 111

- | | | |
|------------------|--------------------------|--------------|
| 15. \$5,347,500. | 17. \$1,095; \$1,047.50. | 19. \$75.16. |
| 16. \$1,058.33. | 18. \$56,550. | |

Page 112

- | | | | |
|--------------|---------------|---------------|-----------------|
| 1. \$81. | 8. \$178.89. | 15. \$7.72. | 22. \$87.92. |
| 2. \$103.20. | 9. \$251.92. | 16. \$38.51. | 23. \$1,433.33. |
| 3. \$79.13. | 10. \$186.92. | 17. \$133.58. | 24. \$100.44. |
| 4. \$257.60. | 11. \$3,825. | 18. \$130.87. | 25. \$389.70. |
| 5. \$349. | 12. \$1,190. | 19. \$144.66. | 26. \$261. |
| 6. \$104.26. | 13. \$120.64. | 20. \$279.29. | 27. \$180.48. |
| 7. \$18.39. | 14. \$12.60. | 21. \$276.11. | 28. \$816. |

Page 113

- | | | |
|--------------|--------------|--------------|
| 1. \$28.67. | 3. \$104.93. | 5. \$55.07. |
| 2. \$245.33. | 4. \$80.06. | 6. \$233.56. |

Page 114

- | | | | | |
|--------------|------------|-------------|-------------|------------|
| 1. \$101.59. | 2. \$7.34. | 3. \$42.73. | 4. \$29.50. | 5. \$3.21. |
|--------------|------------|-------------|-------------|------------|

Page 116

- | | | | |
|------------|-------------|-------------|-------------|
| 1. \$3.17. | 3. \$.97. | 5. \$2.76. | 7. \$15.23. |
| 2. \$4.33. | 4. \$10.77. | 6. \$13.12. | 8. \$1.56. |
1. 1 yr. 6 mo. 3. 3 yr. 5. 3 yr. 7. 1 yr. 4 mo.
 2. $4\frac{1}{11}$ yr. 4. 5 yr. 4 mo. 6. 3 yr. 4 mo.

Page 117

1. $5\frac{1}{2}\%$ 2. 3% 3. 6% 4. 3% 5. 4% 6. 6% 7. 8% 8. 2%

Page 118

1. \$500. 3. \$600. 5. \$450. 7. \$250.
2. \$600. 4. \$400. 6. \$631.17. 8. \$150.

Page 120

6. \$20. 7. \$9.

Page 121

8. \$108. 9. \$230; \$220; \$210; \$60.
10. \$1,776.50; \$1,751; \$1,725.50.
11. \$1,800; \$1,760; \$1,720; \$1,680; \$1,640. 12. \$2,113; \$1.50 less.

Page 122

1. \$1,040. 2. \$10; \$250; \$100. 3. \$212. 4. \$128.13

Page 123

5. \$73.16. 6. \$46.32. 7. \$252.83. 8. \$53.
1. \$55; \$54.88; \$54.75; \$54.63; \$300; \$5,700. 2. \$53.50; \$53.38.

Page 124

3. \$52; \$51.88; \$3,000; \$40; \$39.88. 4. 20 yr.; \$25.13.
6. \$11.80; \$11.75. 7. \$11.20; 3 yr.
1. \$147.70. 2. \$344.50.

Page 125

3. \$670.65. 4. \$608.26.
1. \$139.52. 3. \$491. 5. \$1,244.32. 7. \$1,615.17.
2. \$49.27; \$90.25. 4. \$845.59. 6. \$1,070.94.

Page 126

1. Time; \$100; 3 months' interest; \$.50. Due June 1, 1909, \$100.50.
2. The certificate shown in the Arithmetic would be a *demand* certificate.

Page 127

3. \$2.08. 6. \$.69. 9. \$2,356.36. 12. \$122.60.
4. \$1.08. 7. \$19.44. 10. \$151.25. 13. \$452.81.
5. \$1.33. 8. \$77.78. 11. \$228.38. 14. \$672.51.
15. \$1,505.63.

Page 128

1. \$998.67. 2. \$3,464.51. 3. \$7,965.33.
 4. \$1,880.96. 5. \$19,925.

Page 129

6. \$24.33; \$1,975.67. 7. \$11.83.
 1. \$1,001.94; \$3,507.82; \$8,031.71. 5. \$3.13, discount; \$480.37, proceeds.
 2. \$.56, discount; \$250.69, proceeds.
 3. \$1.52, discount; \$351.69, proceeds. 6. \$9.10, discount; \$614.90, proceeds.
 4. \$2.11, discount; \$402.39, proceeds.
 7. \$10.68, discount; \$523.07, proceeds.

Page 131

1. \$1,151.94. 6. \$1,094.856. 11. \$464.
 2. \$562.43. 7. \$8,344.45. 12. \$2,010.15.
 3. \$703.58. 8. \$8,624.02. 13. \$2,343.32.
 4. \$457.68. 9. \$459.34. 14. \$2,737.14.
 5. \$838.88. 10. \$1,023.25. 15. \$30; \$2,030.
 16. \$30.45; \$2,060.45; \$2,186.89

Page 133

1. \$145.23. 5. \$70. 9. \$99.51. 13. \$83.13. 17. \$300.41.
 2. \$267.05. 6. \$300. 10. \$126.12. 14. \$64.52. 18. \$16,027.40.
 3. \$65.59. 7. \$12,346.67. 11. \$233.55. 15. \$13.90. 19. 1½ yr.
 4. \$51.47. 8. \$28.56. 12. \$15.33. 16. \$5.21. 20. 3¼%.
 21. \$2.55.

Page 134

22. \$70. 26. \$12,424.69. 30. \$164. 34. \$500.07 (\$500).
 23. 5%. 27. \$377.90. 31. \$180,000. 35. \$1,608.75.
 24. \$6,000. 28. 6 yr. 3 mo. 32. \$944.44. 36. \$151.980.
 25. 3 mo. 18 da. 29. 4¼%. 33. 6 da.
 37. (a) \$526,825; (b) \$674,100; (c) \$361,620; (d) \$897,975; (e) \$120,220.
 38. 1½ yr. 39. \$195.85. 40. \$160; \$154.

Page 135

42. (1) \$3.75; (2) \$.60; (3) \$3.67.
 43. (1) \$247.90; (2) \$372.87; (3) \$1,948.10.
 44. \$232.97. 46. \$562.43. 48. 3½ yr.
 45. \$12.55. 47. \$10.15. 49. \$11,980.

Page 136

1. \$30.45.

2. \$31.86.

Page 137

3. \$48.40.

1. \$62.12.

Page 138

2. \$57.39.

1. \$10.58.

Page 139

2. \$50.84.

3. \$3.44.

6. \$132.

7. \$771.45.

8. \$1,586.

Page 140

1. \$287.90.

2. \$90.92.

3. \$181.91.

Page 141

4. \$28.12.

5. \$266.50.

6. \$222.60.

7. \$3,877.47.

Page 146

1. (a) \$1,503.75.

(c) \$3,508.75.

(e) \$10,876.63.

(g) \$1,228.06.

(b) \$1,629.06.

(d) \$86.72.

(f) \$195.99.

(h) \$40.41.

Page 147

1. \$2.95.

2. \$323.37.

3. \$4,488.75.

4. \$1,485.

Page 148

1. \$19.50; \$39.

3. \$309.

5. \$15,500.

2. \$13.50; \$40.50.

4. \$213.75.

Page 149

6. $2\frac{1}{2}\%$.

7. \$1,422.

Page 150

1. \$151.20.

2. \$868.

Page 151

1. $\frac{1}{x^2}$ of 1%; .1%.

2. \$6.25; \$5,768.75.

3. \$160,000.

Page 154

- | | | |
|---|------------------|-------------|
| 1. \$1,787.18. | 3. \$256,250. | 5. 208,125. |
| 2. \$19,218.75. | 4. \$128,562.50. | |
| 6. \$1,782.88; \$19,181.25; \$255,750; \$128,187.50; \$206,875. | | |

Page 155

- | | | |
|---------|-------|---------|
| 1. 7.2% | 2. 5% | 3. 9.5% |
|---------|-------|---------|

Page 158

- | | |
|---|-----------------------------------|
| 1. \$15,768.75. | 3. \$250; \$600; \$800; \$300. |
| 2. \$5,418.75; \$10,625; \$18,800; \$6,915. | 4. \$6.25; \$12.50; \$25; \$7.50. |

Page 160

- | | | | |
|------------------------------|-------------|---------------------|--------------|
| 2. \$36.25. | 4. \$1,190. | 6. 5% | 8. \$55,000. |
| 3. \$18,012.50; \$12,987.50. | 5. \$1,125. | 7. $3\frac{1}{2}\%$ | 9. \$45. |

Page 162

- | | | | | |
|---------------------|---------------------|-------------------|------------------|---------|
| 1. 225. | 5. 64. | 9. 64 cu. in. | 13. .216 cu. in. | |
| 2. $\frac{1}{17}$. | 6. $\frac{1}{17}$. | 10. 125 cu. ft. | 14. 729 cu. ft. | |
| 3. 131. | 7. 6.25. | 11. 1.728 cu. in. | 15. 64 cu. yd. | |
| 4. 625. | 8. .01. | 12. 27 cu. ft. | 16. 343 cu. ft. | |
| 1. 12. | 4. 31. | 7. 25. | 10. 51. | 13. 24. |
| 2. 20. | 5. 17. | 8. 21. | 11. 18. | 14. 13. |
| 3. 16. | 6. 99. | 9. 14. | 12. 60. | 15. 30. |

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|----------------------|----------------------|----------------------|----------------------|---------------|
| 1. 27. | 13. $\frac{1}{17}$. | 25. 41. | 36. 35 yd. | 47. .547. |
| 2. 8.5. | 14. 65. | 26. 55. | 37. 320 rd.; 192 rd. | 48. .447. |
| 3. 1.3. | 15. 150. | 27. $\frac{1}{17}$. | 38. 164 rd. | 49. 2.236. |
| 4. $\frac{1}{17}$. | 16. 28. | 28. 105. | 39. 502. | 50. .141. |
| 5. $\frac{1}{17}$. | 17. 4.5. | 29. 12.5. | 40. 85 ft. | 51. .223. |
| 6. $\frac{1}{17}$. | 18. $\frac{1}{17}$. | 30. 56. | 41. 1.414. | 52. 1.224. |
| 7. 19. | 19. 85. | 31. .45. | 42. 3.316. | 53. 2.645. |
| 8. 20. | 20. 1.5. | 32. 94. | 43. 3.605. | 54. 1.024. |
| 9. $\frac{1}{17}$. | 21. 4.5. | 33. 9.8. | 44. 3.872. | 55. 8.660. |
| 10. $\frac{1}{17}$. | 22. $\frac{1}{17}$. | 34. 41. | 45. 1.732. | 56. .866. |
| 11. 2.1. | 23. $\frac{1}{17}$. | 35. 2.5. | 46. 1.581. | 57. 12.64 rd. |
| 12. 23. | 24. 1.6. | | | |

Page 168

1. 15 in. 2. 13 in. 3. 20 in. 4. Same in each case.
 1. 3. 2. 5. 3. 5. 4. 8. 5. 10. 6. 16.97+ ft. 7. \$238.

Page 169

8. \$84. 10. 45 ft. 12. 50 ft. 14. 50 yd.
 9. 48 ft. 11. 40 ft. 13. 5 mi. 15. 3.6 ft.

Page 171

1. 18.84 in. 3.14 ft. 37.68 in. 18.84 yd. 3. 1 ft. 5. 720.6.
 2. 3.14. 4. 1 in.; $1\frac{1}{2}$ yd.

Page 172

1. 25.12 in.; 50.24 sq. in.
 2. 113.04 sq. in.; 3.14 sq. ft.; 12.56 sq. in.; 12.56 sq. ft.; 314 sq. ft.; 314 sq. in.
 3. Diameter = 4 rd. 6. $9.81\frac{1}{2}$ sq. ft. 9. $\frac{1}{2}$; 9,616 sq. ft.
 4. \$62.17. 7. 1,616.75 sq. ft. 10. 7.74 sq. in.
 5. 3 yd. 8. $12\frac{1}{2}$ sq. ft.

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11. 2.15 sq. in.
 12. 452.16 sq. ft.; 907.46 sq. ft.; 455.30 sq. ft. 13. \$401.92.
 14. 4.5 ft.; 15.896 sq. ft. 15. $\frac{1}{16}$; 2d ring, $\frac{3}{16}$; 3d ring, $\frac{5}{16}$; 4th ring, $\frac{7}{16}$.
 16. \$3,694.18. 17. \$21.98. 18. \$212. 19. $\frac{1}{16}$.
 20. 25,120 mi.; 1,046 $\frac{3}{4}$ mi.; 17.44 mi.

Page 176

1. 125.6 sq. in. 3. 4 in. 5. \$314. 7. \$805.21.
 2. 5 in. 4. 5,652 sq. ft. 6. 6,104.16 sq. ft.

Page 177

1. 40 cu. ft.; 80 cu. ft.; 200 cu. ft. 2. 113.04 sq. ft.; 1,808.64 cu. ft.
 3. 13,564.8 gal. 6. 4 in. 9. \$137.51. 12. 1,130.4 gal.
 4. 4,239 gal. 7. 16 ft. 10. 542.2 cu. yd. 13. 109,795.5 gal.
 5. 56.52 cu. in. 8. 678.24 sq. in. 11. \$46.89.

Page 178

14. 431.75 cu. ft. 15. 11,550 cu. ft. ($\pi = 3\frac{1}{2}$). 16. 28,875 cu. ft.
 17. 48,000 gal.; 336,000 gal.; 17,520,000 gal.
 18. 117,550 gal. 20. 3,679.68+ gal.; \$331.17+; \$478.36.
 19. 432,000 gal.; 157,680,000 gal.; 13.6+ da. 21. 865.46 gal.

Page 180

1. 96 cu. in.
2. 556,516 sq. ft.; 89,042,560 cu. ft.
3. 907,136 sq. ft.
4. $66\frac{2}{3}$ cu. ft.; 324 sq. yd.; $37\frac{1}{2}$ in.; $29\frac{1}{2}$ cu. in.; 1,275 cu. ft.
5. 62,500 cu. ft.
6. 9,918.75 cu. ft.
7. 7 ft.

Page 182

1. 72 sq. in.
2. 301.44 sq. ft.
3. $33\frac{1}{2}$ sq. yd.; \$4.61.
4. 84.78 sq. in.; 75.36 sq. in.

Page 183

1. 12 cu. in.
2. 80 cu. ft.
3. 36 ft.
4. 96 sq. yd.

Page 185

1. 36π sq. in.; 144π sq. in.; 64π sq. ft.; 4π sq. yd.
2. 113.04 sq. in.; 452.16 sq. in.; 200.96 sq. ft.; 12.56 sq. yd.
3. 282.6 ft.
4. \$254.34.
1. 2,144.6656.
2. 1,436.7584 cu. in.
3. 113.0976 cu. ft.; 4.1888 cu. yd.; 33.5104 cu. in.; 268.0832 cu. in.; 523.6 cu. ft.

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4. $r=1$; $d=2$; $v=\frac{4}{3}\pi$.
5. $d=4$; $a=16\pi$; $v=\frac{4}{3}\pi$.
6. $r=3$; $d=6$; $v=36\pi$.
7. $r=4$; $a=64\pi$; $v=2\frac{2}{3}\pi$.
8. $d=\frac{2}{3}$; $a=\frac{4}{9}\pi$; $v=\frac{4}{27}\pi$.
9. $r=\frac{2}{3}$; $a=\frac{4}{9}\pi$; $v=\frac{4}{27}\pi$.
10. $d=2\frac{2}{3}$; $a=\frac{4}{9}\pi$; $v=\frac{4}{27}\pi$.
11. $r=\frac{3}{10}$; $a=\frac{9}{100}\pi$; $v=\frac{27}{1000}\pi$.
12. 268,190,476,190.47+ cu. mi.
13. 6 ft.
14. $42,782\frac{1}{2}$ sq. ft.
15. $314\frac{2}{3}$ sq. yd.
16. \$19.80.
17. $5,749\frac{1}{2}$ cu. in.; 25 gal. nearly.
18. \$15.09.
19. $\frac{2}{3}\pi r^2$; $\frac{1}{3}\pi r^3$.
20. $\frac{1}{3}\pi d^3$; 1,767 $\frac{1}{2}$ cu. in.

Page 187

1. 25 sq. ft.; 100 sq. ft.; 735 sq. ft.; 2,940 sq. ft.; 3,065 sq. ft.
2. 366 sq. in., or $2\frac{1}{2}$ sq. ft.
3. $1\frac{1}{2}$ yd.
4. $8(7\frac{1}{2}\frac{1}{2}\frac{1}{2})$.

Page 188

1. 418 sq. in.
2. 90.2 gal.
3. $21\frac{3}{4}$ sq. ft.
4. 6,225.81 cu. ft.
5. \$103.12.
6. $193\frac{1}{2}$ cu. ft.

Page 189

1. 12.56 sq. in. 2. 3.44 sq. in. 3. 141.3 ft.; 125.6 ft.
 4. 3,179.25 sq. ft.; 2,512 sq. ft.; 667.25 sq. ft.
 5. $8\frac{1}{2}$ in.; 53.38 in.; 226.87 sq. in. 7. 2 ft.; 1 ft.; 6 28 ft.
 6. 7 ft.; 21.98 ft.; 38.47 sq. ft. 8. 6 ft.; 3 ft.; 28.26 sq. ft.
 9. 376 8 sq. in.; 533.8 sq. in.; 942 cu. in.
 10. 8 in.; 207.24 sq. in.; 226.08 cu. in. 12. 2 in.; 69.08 sq. in.
 11. 6 ft.; 150.72 sq. ft.; 207.24 sq. ft. 13. \$ 17.42.

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14. 6,941.76 cu. in. 15. 1 ft. 16. 183.1248 T. 17. $53\frac{1}{2}$ sq. yd.
 18. 80. 21. 20 ft. 24. 12. 27. $1\frac{1}{2}$. 30. 20. 33. $83\frac{1}{2}$.
 19. 20. 22. $\frac{4}{5}$. 25. $\frac{3}{8}$. 28. 4 m. 31. 40. 34. 14.35.
 20. 180 sq. in. 23. 180. 26. 5. 29. 420. 32. 10. 35. 10 d.

Page 191

2. 9×10^6 ; 19×10^9 . 9. 23,045,055. 15. $2\frac{1}{2}$. 21. $\frac{3}{8}$.
 3. 4,436,156. 10. 40,776,810. 16. $11\frac{1}{10}$. 22. $4\frac{1}{8}$.
 4. 51,099,941. 11. 6,356,28572. 17. $1\frac{1}{20}$. 23. $3\frac{1}{2}$.
 5. 3,045,582. 12. 1,018 $\frac{3}{8}$. 18. $23\frac{3}{8}$. 24. $2\frac{1}{8}$.
 6. 294,881. 13. $426\frac{361}{128}$. 19. $2\frac{1}{11}$. 25. $\frac{3}{25}$.
 7. 63,084. 14. $10\frac{484}{81}$. 20. $42\frac{1}{8}$. 26. $10\frac{1}{16}$.
 8. 81.63998.

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27. $\frac{2}{3}$. 34. $1\frac{1}{2}$. 40. 493 mi. 940 ft. 46. 688 gro. 9 doz.
 28. $7\frac{1}{8}$. 35. $\frac{5}{8}$. 41. 128 bu 1 pk. 47. 62 mi. 2760 ft.
 29. $\frac{3}{8}$. 36. $2\frac{1}{8}$. 42. 33 gal. 2 qt. 48. 2 hr. $38\frac{1}{2}$ min.
 30. $37\frac{1}{2}$. 37. $\frac{3}{8}$. 43. 5 yd. 49. 15 yd. $\frac{1}{16}$ ft.
 31. $\frac{1}{2}$. 38. $1\frac{1}{8}$. 44. 27 cu. yd. 16 cu. ft. 50. 3 ft. $3\frac{20}{121}$ in.
 32. $416\frac{3}{8}$. 39. 201 da. 11 hr. 45. 86 A. 40 sq. rd. 51. \$ 12.42.
 33. $1\frac{1}{2}$.

Page 193

52. \$ 13.18. 53. 159 qt.; 333 qt.
 54. 1,296 sq. in.; 720 sq. in.; 14,400 sq. in. 55. 45 yd.; 55 yd. 1 ft. 8 in.
 56. 5 da. 17 hr. 36 min.; 8 da. 8 hr. 65. 3.819 da.
 57. .625. 61. .172. 66. 5.046 mi.
 58. .112. 62. 1.444. 67. 7 gal 2 qt. 1.2 pt.
 59. .583. 63. .115. 68. 45 da.
 60. .281. 64. 9.812. 69. \$ 5.94.; \$ 11.88; \$ 13.93.

Page 194

70. 400 sq. ft.; \$ 100. 77. \$ 24.63. 83. 28.
 71. 18 ϕ . 78. \$ 2.21. 84. Draw lines: 14 in. long, 11 $\frac{1}{2}$ in. long,
 72. \$ 1.80. 79. \$ 7.28. 11 in. long, 11 in. long, 9 in. long,
 73. \$ 1.42. 80. \$ 13.50. 9 in. long, 8 in. long, 7 in. long, 6
 74. \$ 7.22. 81. \$ 7. in. long, 5 $\frac{1}{2}$ in. long.
 75. \$ 58.67. 82. 19.
 76. \$ 12.20.

Page 195

85. \$ 448,923,200.66. 88. 26. 91. 11,000 sq. ft. 94. 39 $\frac{1}{2}$ ft.
 86. \$ 6,002,400. 89. \$ 27. 92. 57.63 mi. 95. \$ 64.75.
 87. .8427 mi.; 4,449.456 ft. 90. 12 ft. 93. 139 $\frac{1}{2}$ ft.

Page 196

96. 244 bbl. 99. \$ 200,000. 102. 166 $\frac{1}{2}$ lb.
 97. 12 $\frac{1}{2}$ %; 16 $\frac{3}{4}$ %; 33 $\frac{1}{2}$ %; 4%. 100. \$ 60,000. 103. 66.59%
 98. 50; 100. 101. 6 gal. .88 pt. 104. 95.4%
 105. 2,975 lb. fish; 4,900 lb. salt meat; 1,500 qt. ice cream; 25 T. potatoes.
 106. 302,034,715 $\frac{11}{16}$ T. 107. \$ 365,355,603 $\frac{1}{2}$.

Page 197

108. \$ 60,000,000. 112. \$ 132.39; \$ 328.50. 115. \$ 18,731.25.
 109. 200. 113. \$ 10; \$ 48.75. 116. \$ 8,259.375.
 110. 141,213 bu.; 350,398 bu. 114. \$ 10; 102.50. 117. \$ 2,896.88.
 111. \$ 105,909.75; \$ 262,798.50.

Page 198

118. \$ 400; 3 $\frac{7}{11}$ %. 122. \$ 180; \$ 144; \$ 120; \$ 102. 125. 331,318,690 $\frac{11}{16}$ bu.
 119. 2,035. 123. 334,400,000 bu. 126. 70,500,000 bu.
 120. \$ 9,095.64. 124. 117,197,105 $\frac{5}{8}$ bu. 127. \$ 6,500,000.
 121. \$ 120; \$ 360.

Page 199

128. 15.6 bu. 132. 37 $\frac{1}{2}$. 134. 1,125 T.
 129. 8,000,000 bu. 133. Great Britain, 60,000,000 bu.; 135. \$ 3 $\frac{1}{2}$.
 130. 252,000 bu. Australasia, 40,000,000 bu.; 136. 300; \$ 18,000.
 131. 12; 1,350. Egypt, 12,000,000 bu. 137. 375,000 lb.

Page 200

138. 268,000,000 bu.; 402,000,000 bu. 143. \$ 50; \$ 16,875.
 139. 3,076,000,000 bu.; 1,538,000,000 bu. 144. .3842 bales; 192 lb.
 140. 2,475,000,000 bu. Yes. 145. \$ 645,480,000.
 141. 504,266 $\frac{2}{3}$ sq. mi. 146. 4,275,151; 39.7%.
 142. 337 $\frac{1}{2}$ bales.

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- | | |
|---------------------------|--|
| 147. 105,000,000 gal. | 152. 10,980,000 bbl. |
| 148. $\frac{1}{18}$. | 153. \$3,111,000. |
| 149. $8\frac{1}{2}\%$. | 154. \$42,000. |
| 150. $724\frac{1}{2}\%$. | 155. 7,825,000 T. |
| 151. 250,000,000 gal. | 156. \$156,600,000; $149\frac{1}{2}\%$. |

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- | | | |
|-------------------------|----------------|---------------------|
| 157. 189,350 sq. mi. | 159. \$24. | 168. 10. |
| 158. Appalachian field, | 160. \$29.46. | 169. 40. |
| 167,941.8 T.; | 161. \$71.875. | 170. 32. |
| Central field, | 162. \$167.33. | 171. 108 sq. ft. |
| 38,755.8 T.; | 163. \$119.60. | 172. 8 in. |
| Western field, | 164. \$171.28. | 173. 1,125 cu. ft. |
| 31,004.64 T.; | 165. 192. | 174. 162 cu. ft. |
| Other fields, | 166. 500. | 175. 876.8 sq. in. |
| 20,669.76 T. | 167. 420. | 176. 118.04 sq. in. |

Page 205

- | | | |
|---|----------------------------|-----------|
| 1. \$12.60. | 4. 30 sq. m.; 24.84 sq. m. | 8. 30. |
| 2. \$3.22. | 5. 536 cm. | 9. 7 m. |
| 3. \$9.60. | 6. 1,600 cm.; 6 cm. | 10. 36 m. |
| 11. 10 dm.; 100 cm. | | |
| 12. 10 mm.; 100 mm.; meter; meter; meter. | 14. 3.937 in.; .03937 in. | |

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- | | |
|---|---------------------------------|
| 15. 11.811 in.; 7.874 in.; 3.937 in. | 17. 1,625 m. |
| 16. 75.692 cm.; 76.454 cm.; 72.644 cm. | 18. 84 mi. (1,625 km. = 1 mi.). |
| 1. 200 sq. cm.; 40,000 sq. cm.; 1,000,000 sq. mm. | |

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- | | | |
|--------------------------|----------------------|---------------|
| 2. 100 sq. m.; 200 ares. | 5. 74.1 A.; 123.5 A. | 8. \$540. |
| 3. 100 ha. | 6. 100,000 sq. m. | 9. 60,500. |
| 4. \$3,000. | 7. 4,800. | 10. 181.5 ha. |

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- | | | |
|--|-----------------------------------|------------------------------|
| 1. 346.5 metric tons. | 3. 63. | 5. 26.25 steres; 7.25 cords. |
| 2. 57.2 cu. m. | 4. 6,000 cu. cm. | 6. \$3,071.25. |
| 7. 50 cu. m.; 1,768.5 cu. ft. | 8. 52,296 cu. cm. in excess; 52¢. | |
| 9. 3.1546 m.; 1.06155 m.; .8189 m.; 2.73+ cu. m. | | |
| 10. 3.62 steres. | | |

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1. 405 l. 2. 1000 l. 3. 400 hl. 4. 50%. 5. \$ 250. 6. 1,100 l.; 1,166 qt.

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7. 11,000,000 cu. cm.; 11 kl.; 2,915 gal. 9. 2,189.9 l.; 2,321.29 qt.
8. 5,000. 10. $\frac{1}{2}$; $\frac{1}{16}$; $\frac{1}{128}$.

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1. 500 g. 7. 1,540 lb.
2. 1000 g.; 1 kilo. 8. .05 g.; .3 g.; .35 g.
3. 830 g.; .83 kilo, 1.826 lb. 9. .643 kilos.
4. 5 metric tons; 5.5 English tons. 10. .9508 hectograms.
5. 1000 l.; 1000 kilos; 2,200 lb. 11. 28,125 g.; 28.125 kilos.
6. 184 kilos; 404.8 lb. 12. 2,000 cu. ft.; 125,000 lb.; 56,250 kilos.

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2. $1\frac{3}{4}$ sq. yd. 5. $7\frac{1}{2}$. 8. 8.8 kilos. 11. 84,780 kilos.
3. \$ 1,706.25. 6. 984 ft. 9. 30 sq. m.; 3,000 tiles. 12. 185.64 m.
4. 11,000 lb. 7. 196,020 lb. 10. .09051 g.

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1. \$ 976 2. £ 205 2s. 7d. 3. \$ 6,930.91. 4. \$ 2,350,000. 5. £ 13,555,000.
6. \$ 492,480,000; \$ 65,960,000; \$ 16,950,000; \$ 67,775,000; \$ 643,165,000.
7. \$ 479,330,784; \$ 64,198,868; \$ 16,497,435; \$ 65,965,407.50; \$ 625,992,494.50.
8. Denmark, \$ 75.80; Switzerland, \$ 63.32; Germany, \$ 36.64; Norway, \$ 36.16;
Australia, \$ 35.62; United States, \$ 30.39; Austria-Hungary, \$ 26.36;
France, \$ 21.57; Great Britain, \$ 20.07; Canada, \$ 11.01.

	IMPORTS	EXPORTS
9. British Possessions . . .	\$ 483 895,342 00	\$ 405 996,332 41
United States	618 481 477 31	71 617,793.72
France	250 122 493.18	66,701.446 16
Germany	138 861,484.77	109,622,715.74
Others	802.736,576 95	481.708,088.71
Total	\$ 2,289,097,374.16	\$ 1,135,646,376.74

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1. 171.39 fr. 2. 84.58 fr. 3. 76.71 fr. 4. 2,575 fr. 5. \$ 388.07.

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6. 79 fr. 7. \$ 9.20. 8. \$ 87.91. 9. 49.21 fr.
10. United States: Imports, 10.4+%; exports, 6.2+%.
Others: Imports, 2.868 million francs; exports, 2 126 million francs.

	IMPORTS	EXPORTS
11. British Isles	\$ 129,117,000	\$ 239,127,000
United States	88,780,000	49,022,000
Germany	79,516,000	88,780,000
Others	553,524,000	410,318,000
Total	\$ 850,937,000	\$ 787,247,000

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1. 3.87 M. 2. 9.425 M. 3. 63.36 M. 4. 1.98 M.
5. (1 M. = 25¢). (1) \$.1125; \$.967 +. (2) \$.1625; \$2.356 +. (3) \$.132;
\$.15.84. (4) \$.825; \$.495 +. 6. \$4,775. 7. 5,013.05 M.
8. (1 M = 25¢) \$166,462,644; \$43,051,993.75; \$88,977,512.50;
\$19,981,402.50; \$146,589,602.
9. Army, \$159,596,059.94; navy, \$41,276,099; postal and telegraph,
\$85,307,190.11; railways, \$19,157,169.65; administration, \$140,542,780.92.

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1. Hats, \$1.44; gloves, \$.74; suit, \$16.25; overcoat, \$12.63; necktie, \$.36.
2. £18 14s. 2d.; 1st item, \$11.56; 2d item, \$25.35; 3d item, \$41;
4th item, \$15.63.
3. \$91.41. 4. 7s. 6d. 5. \$50. 6. \$3.10. 7. \$13.40. 8. \$325.
9. \$60; \$57.12; \$79.97. 10. \$20,000. 11. 175.07 + M.

Page 220

1. \$49.65. 2. \$49.09. 3. \$49.46.
4. £102 10d.; 2,562.50 fr.; 2,062.50 M. 5. \$49.76.

Page 223

1. $11a + 14b$. 8. $50m + 5n + 2p$. 15. $a + 2c$.
2. $a + m + b + 7c$. 9. $10m + 10n + 10p$. 16. $5a + 5b$.
3. $32a + 8b$. 10. $4a + 5b$. 17. $20 + 6p$.
4. $10a + 12b$. 11. $4a + 3b$. 18. $x + 6y + 7z$.
5. $6a + b + 2c$. 12. a . 19. $40m + 2n + 10p$.
6. $9a + 3b + c$. 13. $8b$. 20. $4a + b + 2c$.
7. $3m + 6n + 10p$. 14. $a + 4b + 3c$. 21. $8m + 5n$.

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22. $(2a + 3b + 4c)$ bu. 24. $(125a + 25b + 22c)$ ft.
23. $(4a + b + 3c)$ bu.; $(6a + 4b + 7c)$ bu. 25. $(2m + 5n)$ bu.
1. 3^2 . 4. 2^3a^2 . 7. b^2 . 10. s^4 . 13. $2^2 \times 5^2$.
2. 2^3 . 5. 5^2 . 8. 2^4b^3 . 11. c^2 . 14. a^2y^2 .
3. 3^4 . 6. 2^4 . 9. 7^2 . 12. 3^3 . 15. $2^3 \times 5^2a^3b^3$.

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|--------------|---------------|---------------|---------------|----------------|
| 16. 7^2 . | 19. 60^2 . | 22. $12a^2$. | 25. $18b^4$. | 28. $8x^5$. |
| 17. 12^2 . | 20. 12^2 . | 23. $40m^2$. | 26. $24c^2$. | 29. $120s^2$. |
| 18. 12^2 . | 21. 100^2 . | 24. $10b^2$. | 27. $8x^2$. | |
-
- | | | |
|-----------------------------|---------------------------|----------------------------|
| 1. $3a^2 + ax + 3ab + bx$. | 5. $n^2 + 4n + 4$. | 9. $ab + bx + ay + xy$. |
| 2. $x^2 + 2x + 1$. | 6. $2 + 2m + m^2 + m^2$. | 10. $3y^2 + 11y + 6$. |
| 3. $s^2 + 9s + 18$. | 7. $2b + ab + 2x + ax$. | 11. $x^2 + 2xy + y^2$. |
| 4. $4ax + 5a + 4x^2 + 5x$. | 8. $x^2 + 10x + 25$. | 12. $x^2 + x^2 + 2x + 2$. |

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- | | | |
|-------------------------|---------|------------|
| 14. $a^2 + 2ar + r^2$. | 15. 32. | 16. 1,849. |
|-------------------------|---------|------------|

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|------------------|------------------|---------------|---------------|
| 1. $c + d$. | 3. $m + p + q$. | 5. $a + 2b$. | 7. $x + 3y$. |
| 2. $m + p + q$. | 4. $2a + b$. | 6. $a + 2b$. | 8. $m + 2n$. |

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- | | | |
|-----------------------|-----------------|--------------------------------------|
| 1. 25; 105; $32b^4$. | 5. $16xy$. | 2. $16a^2b^3$; $55a$; $18a^2b^2$. |
| 3. $25mp^2$. | 6. $19p^4q^4$. | 7. $12s^4t^2$. |
| 4. $35a^2b^2$. | | 8. $14m^2n^2$. |
| | | 9. $65mn^5$. |
| | | 10. $21xy^6$. |

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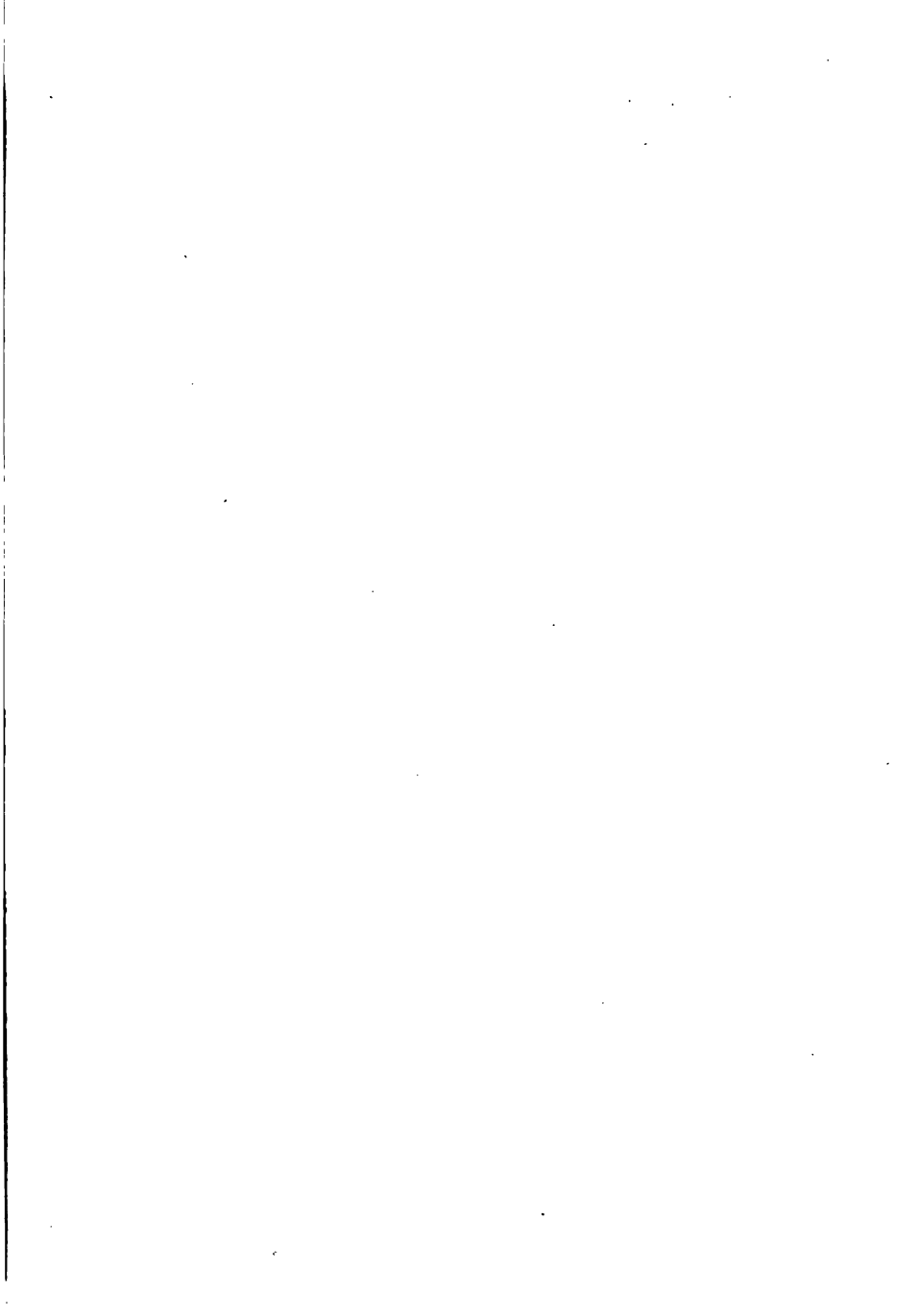
- | | | | |
|-------------|--------------|-----------|-------------|
| 1. 7. | 3. \$171.43. | 5. \$500. | 7. \$1,200. |
| 2. \$2,000. | 4. \$50. | 6. \$400. | 8. 30 ft. |

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- | | | |
|---------------|---------------------------|--------------------------------------|
| 9. \$2,000. | 13. \$4,000. | 16. 21ϕ ; $25\frac{1}{2}\phi$. |
| 10. \$1,500. | 14. $27\frac{1}{2}$ A. | 17. $3(2c - \$100) = h$. |
| 11. 5. | 15. $79\frac{1}{2}\phi$. | 18. \$1,800. |
| 12. \$11,000. | | |

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- | | | | |
|-----------------------------|-------------------------|--------------------------|------------------------|
| 1. $2a$. | 7. $66x^2$. | 13. $5x + 18y$. | 19. $7b + 5c$. |
| 2. 10. | 8. 20. | 14. $2a + 17b$. | 20. $(5a)^2$. |
| 3. y . | 9. $8a + 3b$. | 15. $3 + 6a$. | 21. $(3x)^2$. |
| 4. $15x$. | 10. $48y^2 + 3y$. | 16. $9m + 15$. | 22. $(16)^2a$. |
| 5. $3ay^2$. | 11. $2a + 3b$. | 17. $8q$. | 23. $(36)^2x$. |
| 6. $85a + 3$. | 12. $11a + 11$. | 18. $x + 9q$. | 24. $6x^2 + 11x + 4$. |
| 25. $30a^2 + 21ab + 3b^2$. | 26. $a^2 + 2am + m^2$. | 27. $2a^2 - 3ac + c^2$. | |
| 28. $56x$. | 30. $96a$. | 32. 6. | 34. 2. |
| 29. $168m$. | 31. a^2x^4 . | 33. 5. | 35. 4. |
| | | | 36. 20. |
| | | | 37. 5. |





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